SHIP ROLL CONTROL BY PUMP ACTIVATED TANKS

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SHIP ROLL CONTROL BY PUMP ACTIVATED TANKS

by

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FORTRAN subroutines for the design of passive and pump-activated U-tube anti-roll tanks are developed, and the effects of a tank installation on the

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Ship Roll Control by Pump Activated Tanks

by

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FORTRAN subroutines for the design of passive and pump-activated U-tube anti-roll tanks are developed, and the effects of a tank installation on the ship parameters is observed. Many controller design methods are investigated, and final design is accomplished by exhaustive search of closed-loop frequency responses. Frequency responses of passive and activated tank stabilized ships are compared with that of an unmodified ship to illustrate the effectiveness of design. The effects of vessel displacement variations due to consumption of fuel and stores on tank effectiveness and controller design are discussed.

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I. FORMULATION OF THE PROBLEM

A ship on the ocean is in constant motion. Ideally, the only motions would be translation in the forward direction and rotation about the vertical axis, associated with propelling and maneuvering the ship. Realistically, there are perturbation motions in addition to the desired motions in the form of translations along and rotations about the longitudinal, transverse, and vertical axes of the ship. The purpose of this thesis was to study the rotation about the longitudinal axis, known as roll, and arrive at a scheme for controlling it. A procedure for designing the installation of a U-tube anti-roll tank system was consolidated from several sources [Chadwick, 1950] [Chadwick and Klotter, 1953][Blagoveshchensky, 1962][Vachanaratana, 1973][Ziegler, 1975] and presented in the form of a FORTRAN subroutine. Many controller design methods were investigated, and the final design was accomplished by exhaustive search of closed-loop frequency responses.

A. WAVESLOPE SPECTRUM OF THE OCEAN.

Before a meaningful analysis of ship motions in water could be conducted, the motions of the water itself had to be understood. Because a ship tends to ride normal to the surface of the water, the most significant physical property of waves with respect to ship motions is the waveslope, ψ. At the crest and in the trough of a wave, the waveslope is zero, while at some point nearly midway between those extremes, the waveslope is maximum.

Several theories of wave motion have been advanced, including the simple harmonic wave; the Stoke's Wave, which

is a harmonic wave with a second harmonic added; and the trochoidal wave. Geometrically, a trochoidal curve is constructed by rolling a circle of radius R under a horizontal line, and tracing the locus of a point a radial distance $r<R$ from the center of the circle. The trochoidal curve may be steep or shallow depending on the ratio of r:R, and corresponds very closely to actual observed ocean waves, where $2\pi R = L$ = wavelength, and $2r = h$ = trough-to-crest wave height.

Ocean waves are primarily caused by the transfer of kinetic energy from the wind, which is the result of the transfer of thermal energy in weather phenomena. As the wind blows over a flat surface of water, the water particles at the air-sea interface are excited by frictional forces, and tend to move downwind. The freedom of motion of these particles is limited by the fluidic properties of water, and thus they tend to accumulate in successive ridges, or ripples, on the sea surface. Once these ridges are formed, further wave buildup continues by a different mechanism. On the upwind side of a ridge, a small local high pressure region develops which exerts a downward force on the water surface, forming the wave trough. On the downwind side of a ridge, a corresponding small local low pressure region develops which draws the water surface upward, forming the wave crest. The wave continues to grow as long as the wind continues to supply energy in excess of that consumed by the internal friction of the water in the wave, which it does until the wave velocity equals the wind velocity.

Three key factors govern the development of wave systems. They are the initial difference between the velocities of the wind and any existing wave systems; the length of time (duration) that the wind continues at a given velocity; and the fetch, or distance of relatively uniform depth water over which the wave will run before being

altered by a change of depth or destroyed by encountering a land mass. (The passage of a vessel through an ocean wave system does little to alter the growth or shape of the system.)

The rate of increasing wave height is a function of the initial difference between the velocities of the wind and the existing wave system. A sudden change of wind velocity, such as that associated with the passage of a front, can cause the waves to increase at a rate of one or two feet per minute. There is a physical limit to the increase in wave height, and it is defined in terms of the wave height to wave length ratio, $h:L=1:7$. Near this ratio limit, excess wind energy blows off the tops of the waves, forming turbulence known as "white caps."

The rate of increasing wave length generally lags behind the rate of increasing wave height. Thus in regions of short fetch, a sudden change of wind velocity may never produce waves with long wavelengths due to the limiting ratio. In contrast, on the open ocean where there is a long fetch, a change of wind velocity can produce long waves of relatively low height. For waves of length $L>100$ feet, there is a diminishing likelihood of ever approaching the maximum value of the ratio. In practice, for wavelengths between 500 and 500 feet, the ratio rarely exceeds 1:20; for the longer ocean waves (of length over 1000 feet), the ratio drops to 1:50.

The above discussion of wave development only hints at the complexity of attempting to define the waveslope spectrum of the ocean. Conferences have been held, and many books have been written on the subject of ocean spectra. [Conference, Easton, Maryland, 1961] [Inoue, 1966] [Comstock, 1967] [Gillmer, 1969] [Ziegler, 1975] Many ocean spectra plot relative wave energy as a function of frequency

with "sea state" as a parameter. Others define wind velocity and duration as parameters. Still others define "significant wave height" as a parameter, with many definitions of "significant wave height" itself. Spectra have been constructed for given locations citing mean wave height, or mean squared wave height, or RMS wave height as parameters.

Two other factors which affect the effective ocean spectra are vessel speed and angle of encounter. To illustrate these effects on the ocean spectrum, consider a ship proceeding with velocity identical to that of a single-frequency wave train. If the ship heads directly into the approaching wave train, the apparent frequency of the wave doubles, but the roll coupling with the ship goes to zero. When the ship travels parallel to the wave train, which is to say the ship is in a beam sea, the apparent frequency of the wave is the same as the actual frequency of the wave, and the roll coupling is maximum. When the ship heads in the same direction as the wave train ("following seas" condition), the apparent frequency of the wave drops to zero, as does the roll coupling. If B denotes the ship's angle of encounter with the wave train, where $B=0^\circ$ for a following sea, $B=90^\circ$ for a beam sea, and $B=180^\circ$ for a head sea, then the apparent waveslope is related to the actual waveslope, and the apparent frequency of the sea is related to the actual frequency by the expressions [Ziegler, 1975]:

$$\psi(B) = \psi_{max} \sin B \tag{1}$$

$$\omega(B) = \omega_{sea} (1.- \omega_{sea} u_{ship} /g) \cos B \tag{2}$$

u_{ship} = forward velocity of the ship

g = acceleration due to gravity

Since the ultimate reason for this study was to aid in

the design of a general-purpose anti-roll stabilizer, the ocean spectrum chosen was a wave height power density spectrum, with sea state and significant wave height (1/3 highest waves) as parameters. The waveslope spectrum was then computed on the basis of the trochoidal model of the ocean, where

$$T = 1/f = 2\pi/\omega \quad \text{seconds} \qquad (3)$$

$$L = 5.118\ T^2 \quad \text{feet} \qquad (4)$$

$$\psi_{max} = (h/L) \times 57.3 \quad \text{degrees (first approximation)} \qquad (5)$$

B. SHIP ROLLING IN WAVES.

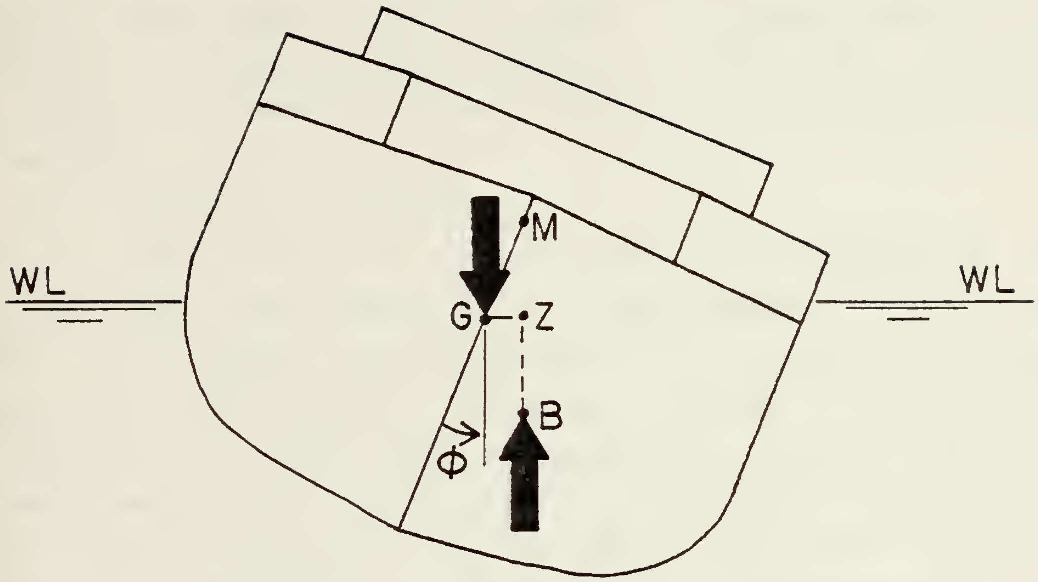
A ship on the ocean moves, in general, with six degrees of freedom relative to the coordinate axis system. By convention, the positive x-direction is forward along the longitudinal centerline of the ship, the positive y-direction is horizontally outward on the starboard beam, and the positive z-direction is vertically downward. The center of the coordinate axis system is located at the vessel center of gravity. Rotation about the x-axis is known as roll, with positive roll angle, ϕ, defined as an inclination to starboard. Roll can be considered separately from the other degrees of freedom, which reduces the study of ship motions to a two-dimensional problem with one degree of freedom.

1. Definitions and Concepts of Naval Architecture.

The following definitions and concepts of naval architecture [Comstock, 1967] [Gillmer, 1969] are given for the benefit of the reader, and are illustrated in Figure 1-1.

a. Displacement.

The displacement of a vessel, Δ, is defined as the weight of the volume of water displaced by the vessel.



G = Center of Gravity
B = Center of Buoyancy
M = Metacenter
GM = Metacenter Height
GZ = Righting Arm
ϕ = Ship Roll Angle Relative to Earth Reference
WL = Water Line

Figure 1-1. Definitions and Concepts of Naval Architecture

Due to the Principle of Archimedes, the displacement equals the weight of the vessel itself. Displacement is always given in long tons, where one long ton equals 2240 U.S. pounds. The weight of the water displaced equals the weight of the vessel, independent of the density of the water. The volume of water displaced, however, is a function of the density of the water. For purposes of most calculations, the density of fresh water is given as 36 cubic feet per ton, whereas the density of salt water is given as 35 cubic feet per ton.

b. Center of Gravity.

The center of gravity, G, is defined as the point at which the summation of the moments of all the weights in a vessel with reference to any axis passing through that point equals zero. The center of gravity of most warships of usual form is generally located near the designer's waterline, in or near the plane of the midship section. For purposes of buoyancy and stability analysis, the entire weight of a vessel is considered to be concentrated at, and the force of the weight acts vertically downward through, the center of gravity.

c. Center of Buoyancy.

The center of buoyancy, B, is defined as the geometric center of the underwater form of the vessel. The force of buoyancy, equal in magnitude to the force of the displacement of the vessel, acts vertically upward through the center of buoyancy. When a vessel is in static equilibrium, the center of buoyancy is located directly under the center of gravity, such that the force vectors of displacement and buoyancy are equal and opposite. Due to this definition of the center of buoyancy, as the condition of a vessel varies from that of static equilibrium, the location of B varies also.

d. Metacenter.

As a ship inclines, the shape of the underwater form changes, and the center of buoyancy migrates to remain at the geometric center. The migrations form a locus which is dependent upon and directly related to the shape of the ship's hull. This locus is generally elliptic and as such cannot have a fixed center, but for an infinitesimal increment of the locus the instantaneous center, called a metacenter, can be located. The locus of metacenters is geometrically an evolute, and is called the metacentric. For small angles of inclination, up to 10° for most ships, the metacenters are co-located at a point known as the Metacenter, M.

e. Metacentric Height.

When a ship is upright, that is, when the angle of inclination $\phi = 0^\circ$, the centers M, G, and B lie in a straight vertical line over the keel. The distance between G and M is known as the metacentric height, GM. If M is above G, GM is defined as being positive.

f. Righting Arm.

When a ship is inclined, the center of buoyancy is no longer on the original vertical centerline. A point Z is described by the intersection of a line drawn vertically upward from B with a line drawn horizontally away from G. The distance GZ is known as the righting arm, and for small angles of inclination ϕ,

$$GZ = GM \sin\phi = GM\,\phi \qquad (6)$$

g. Righting moment.

The righting moment, M_r, is the product of the force (displacement) and the righting arm:

$$M_r = \Delta\, GZ = \Delta\, GM\,\phi \qquad (7)$$

h. Metacentric Height and Stability.

The metacentric height is a measure of the statical stability of a vessel. A large positive GM corresponds with a pair of high-frequency complex conjugate

poles in the left half of the s-plane, far removed from the $j\omega$-axis. A small positive GM corresponds with a pair of low-frequency complex conjugate poles in the left half of the s-plane, close to the $j\omega$-axis. If GM = 0, then there are two poles at the origin of the s-plane. A negative GM puts poles in the right half of the s-plane.

2. Equations of Motion.

a. Natural Period of Roll.

In still water, a ship can be made to roll by applying and removing external heeling moments. In the inclined position, the righting moment is equal and opposite to the heeling moment, and the potential energy of the ship is equal to the work done by the external force causing the inclination. When the external moment is removed, the righting moment produces rotation of the ship towards the upright position. The potential energy is converted to kinetic energy such that when the ship is in the upright position, assuming unresisted roll, all the potential energy has been converted to kinetic energy. As a result, the ship will continue to rotate past the upright position until all kinetic energy has been reconverted to potential energy, at the opposite limit of roll. Assuming no loss due to friction, the ship would continue to oscillate, or roll, indefinitely with constant amplitude. The period of these oscillations can be found from simple harmonic motion relationships to be [Comstock, 1967]

$$\begin{aligned} T_\phi &= (2\pi k)/\sqrt{gGM} \\ &= (1.108k)/\sqrt{GM} \\ &\approx (0.44b)/\sqrt{GM} \end{aligned} \tag{8}$$

where

g = acceleration due to gravity, ft/sec^2

k = radius of gyration of mass of ship, feet

b = extreme beam of ship, feet

b. Roll by Wave Action.

The cause of ship rolling in a seaway is primarily the unbalanced moments which result from the center of buoyancy shifting in response to the change of the waterplane, and thus the shape of the underwater portion of the hull, as waves pass under the ship. A ship will usually attempt to ride normal to the surface of a wave. A ship also moves in a circular orbit as do the particles of water in a wave. The orbits induce centrifugal forces, which are opposed by the dynamic forces in water.

Rolling in waves is a forced oscillation, and the ship will tend to oscillate at or near the frequency of the waves. If the waves are not uniform, which is usually the case, a ship will attempt to oscillate with its natural frequency. The overall roll, therefore, is a combination of the ship's natural and the wave's periods, with the wave influence being generally more predominant.

c. Equations of Motion.

A ship can be modeled as a second-order, spring-mass-damper system. [Chadwick, 1950] [Comstock, 1967] The spring constant, or static righting coefficient, was discussed earlier in this paper in the definition of the righting moment. The static righting coefficient is

$$K_s = \Delta GM \tag{9}$$

The inertia term, or roll inertia coefficient, is the product of the mass of the ship and the radius of gyration of the mass of the ship squared:

$$J_s = (\Delta/g)\, k^2 \tag{10}$$

The radius of gyration is found by timing the natual

period of roll in protected (still) water, and massaging the equation for roll period as follows:

$$k = (T_\phi \sqrt{gGM})/2\pi$$
$$= (T_\phi \sqrt{GM})/1.108 \quad (11)$$

The roll damping coefficient is derived from the observation that the damping ratio of a typical ship is approximately

$$\zeta_{ship} = 0.1/\pi = 0.0318 \quad (12)$$

By working backwards through the second-order system relationships, the roll damping coefficient was found to be

$$B_s = 0.1\ (T_\phi \Delta GM)/\pi^2 \quad (13)$$

On the forcing function side of the equation of motion, following the classical usage of Froude and Krylov, [Chadwick and Klotter, 1953] the ship-sea coupling coefficient was considered to be approximately equal to the static righting coefficient. Therefore, the equation of motion for a ship in a seaway is:

$$J_s \ddot{\phi} + B_s \dot{\phi} + K_s \phi = K_s \psi \quad (14)$$

Dividing through by the roll inertia coefficient and performing the LaPlace Transform operation, the system transfer function was obtained:

$$\left[s^2 + (B_s/J_s)s + (K_s/J_s)\right]\Phi(s) = (K_s/J_s)\ \Psi(s) \quad (15)$$

$$\left[s^2 + (2\zeta_s\omega_s)s + \omega_s^2\right]\Phi(s) = \omega_s^2\ \Psi(s) \quad (16)$$

$$\frac{\Phi(s)}{\Psi(s)} = \frac{\omega_s^2}{s^2 + (2\zeta_s\omega_s)s + \omega_s^2}$$

$$= \frac{(K_s/J_s)}{s^2 + (B_s/J_s)s + (K_s/J_s)} \tag{17}$$

It was useful in the analysis to normalize the equations such that the ship's natural frequency was unity. This was accomplished in the following manner:

Starting with the general equation of motion, divide through by the static righting coefficient:

$$(J_s/K_s)\,\ddot{\phi} + (B_s/K_s)\,\dot{\phi} + \phi = \psi \tag{18}$$

Let $t' = \omega_s t = (\sqrt{K_s/J_s})\,t$

Define $d\phi/dt = \dot{\phi}$

$d\phi/dt' = \phi'$

$d^2\phi/dt'^2 = \phi''$

Then $\dot{\phi} = (\sqrt{K_s/J_s})\,\phi'$

$\ddot{\phi} = (K_s/J_s)\,\phi''$

The equation was then rewritten as

$$\phi'' + (B_s/\sqrt{K_s J_s})\,\phi' + \phi = \psi \tag{19}$$

The LaPlace Transform operation was performed, and the system transfer function (normalized) was obtained:

$$\left[s^2 + (B_s/\sqrt{K_s J_s})s + 1\right]\Phi(s) = \Psi(s) \tag{20}$$

$$\frac{\Phi(s)}{\Psi(s)} = \frac{1}{s^2 + (B_s/\sqrt{K_s J_s})s + 1} \tag{21}$$

3. <u>Frequency Response.</u>

In order to study the frequency response of a vessel to the waveslope spectrum of the ocean, it was necessary to introduce a numerical example. The example chosen was a

well used example [Blagoveshchensky, 1962], which was:

L = 49.1 meters (161.0 feet)

b = 9.0 meters (29.5 feet)

Δ = 936.0 metric tons (921.2 long tons)

GM = 0.73 meter (2.4 feet)

T_{ϕ} = 8.63 seconds

The radius of gyration of the mass of the ship, and the coefficients of the equation of motion were computed as follows:

$$k = (8.63\sqrt{(9.81)(0.73)})/2\pi = 3.68 \text{ meters}$$

$$J_s = (936/9.81)(3.68)^2 = 1289.021 \text{ tons-meters-sec}^2$$

$$B_s = (0.1)(8.63)(936)(0.73)/\pi^2 = 59.7 \text{ tons-meters-sec}$$

$$K_s = (936)(0.73) = 683.28 \text{ tons-meters}$$

These values were then substituted into the transfer function, the roots of the characteristic equation were found, and the frequency response was determined.

$$\frac{\Phi(s)}{\Psi(s)} = \frac{0.53}{s^2 + 0.046s + 0.53} \qquad (22)$$

roots (poles) at $s = -0.02 \pm j\,0.73$

natural frequency $\omega_s = 0.73$

damping ratio $\zeta_s = 0.032$

The frequency response is graphically illustrated in Figure 1-2. The Digital Simulation Language (DSL/360) program which generated the plot is listed in Appendix A. The zero decibel low frequency response demonstrates the natural tendency of a ship to ride normal to the surface of a wave. The resonant peak which occurs at 0.724 rad/sec has a gain of 23.9 db, or a magnification factor of 15.6. According to the waveslope spectrum of the ocean, tabulated

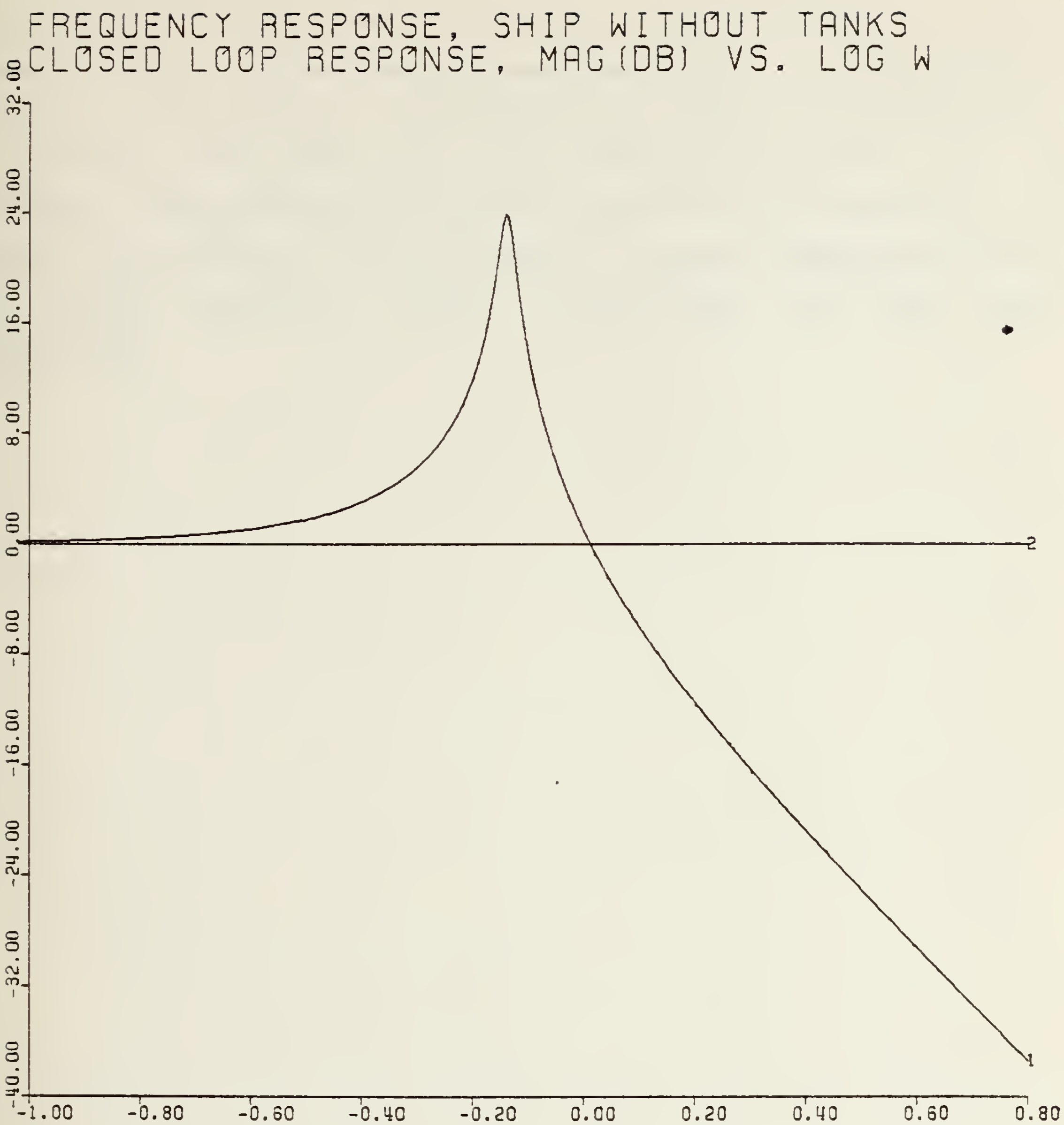


Figure 1-2. Frequency Response of Typica Ship

in Appendix E, for a sea of 15 foot significant wave height (sea state 6) the maximum waveslope angle at 0.724 rad/sec would be 2.23 degrees. Therefore, the ship would experience a maximum roll angle of 34.8 degrees, relative to the horizon. This severity of roll causes crew discomfort, and limits the ability of the ship to conduct operations, such as gunfire support, flight operations, or small boat operations.

II. SHIP STABILIZATION BY PASSIVE ANTI-ROLL TANKS

A. DEVELOPMENT OF THE EQUATIONS OF MOTION

In order to apply servo theory to the problem of ship stabilization, the physical system was assumed to be linear, with lumped parameters and constant coefficients. It was also assumed that the rolling degree of freedom could be considered independently of the other degrees of freedom, such that the ship oscillates as if about a fixed center of rotation. The problem thus became a two-body problem, the ship and the tank-fluid, each with one degree of freedom. The tank-fluid was idealized as a one-dimensional filament, with provision for varying cross-section but with constant velocity across each cross-section. These assumptions required small angles of ship roll but allowed for moderate angles of tank-fluid level, both being reasonable in a stabilized ship.

1. Double Pendulum Analogy.

As discussed earlier in this paper, a ship at sea behaves like a damped oscillator, and was modeled as a second-order spring-mass-damper system. Another appropriate model which was considered was the pendulum model [Chadwick, 1950] [Chadwick and Klotter, 1953], with a spring constant, an inertia constant, and a damping constant.

The stabilizer chosen was a U-tube anti-rolling tank system. The water or other fluid in the U-tube acts as a hydraulic pendulum. Since the tank was to be contained within and wholly supported by the ship, the ship-tank system that resulted was a pendulum supporting a pendulum, or a double pendulum. In general, this kind of system leads

to two second-order differential equations with coupling terms.

2. Nomenclature.

There are two types of nomenclature needed here. These are the parameters and variables of the differential equations, and the dummy variables and parameters which characterize the tank geometry.

a. Differential Equations.

The differential equations are second-order in nature, and have forcing functions. The variables and parameters are as follows:

Dependent variables of the forcing side:

ψ = instantaneous effective waveslope

Parameters of the forcing variables:

K_{ss} = ship-sea coupling coefficient

Torques produced on the forcing side:

$K_{ss}\psi$ = disturbance torque due to waves

Dependent variables of the homogeneous side:

ϕ = ship roll angle relative to the horizon

θ = tank-fluid level angle relative to the ship

Parameters of the homogeneous variables:

Ship self-terms:

J'_s = roll inertia coef., unmodified ship (no tanks)

B'_s = damping coef. of unmodified ship

K'_s = static righting coef. of unmodified ship

J_s = roll inertia coef., modified ship (with tanks)

B_s = damping coef. of modified ship

K_s = static righting coef. of modified ship

<u>Tank self-terms:</u>

J_t = equivalent inertia coef. of tank-fluid

B'_t = skin-friction damping coef. of tank-fluid

B_t = overall damping coef. of tank-fluid

K_t = equivalent static coef. of tank-fluid

<u>Mutual coupling terms:</u>

J_{st} = mutual-coupling inertia coefficient

B_{st} = mutual-coupling damping coefficient

K_{st} = mutual-coupling static coefficient

It is a feature of the double pendulum that

$$K_{st} = K_t$$

$$B_{st} = 0$$

As discussed earlier in this paper

$$K_{ss} = K_s$$

b. Tank Geometry.

The notation used in the derivation which depends upon tank geometry is shown in Figure 2-1, and is given below.

s = a generalized coordinate, the curvilinear distance along the fluid trajectory. $s=0$ at the tank-fluid surface in the starboard tank, and $s=S$ at the tank-fluid surface in the port tank, when $\theta = 0^\circ$.

R = lever arm of tank, the horizontal distance from the vertical centerline of either side tank to the vertical centerline of the ship.

$r(s)$ = the radial distance from the virtual center

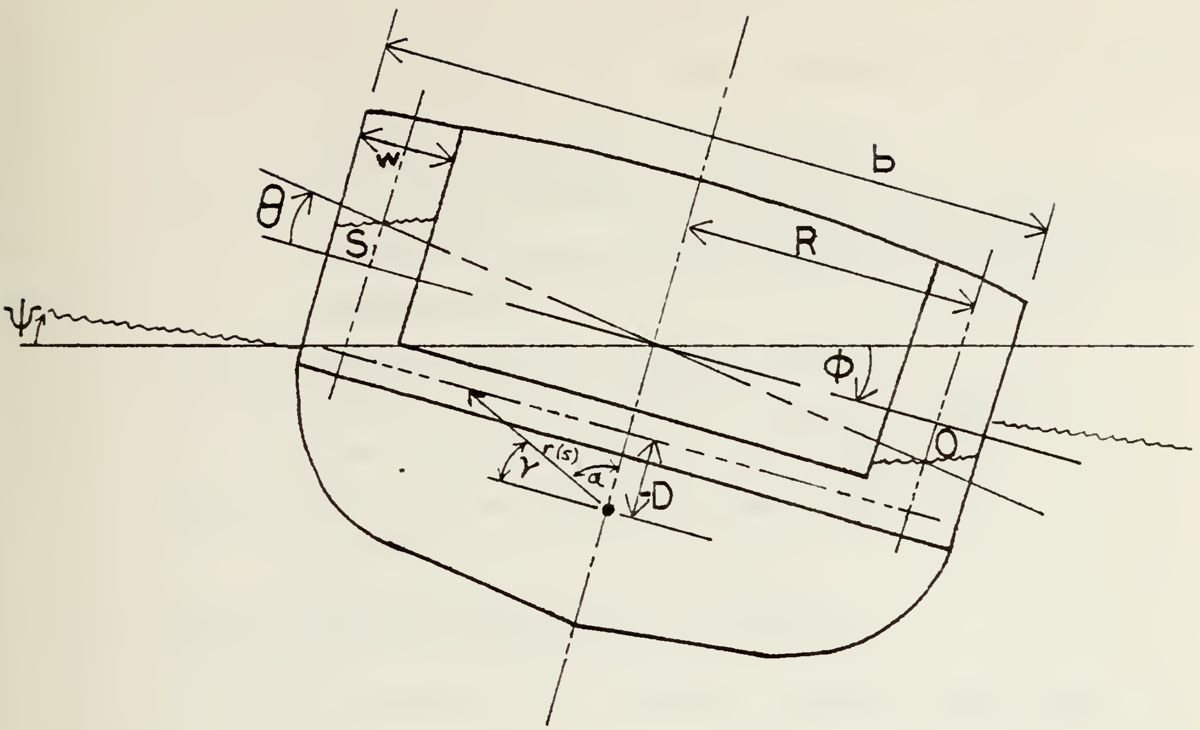


Figure 2.1. The Dependent Variables and Nomenclature of Tank Geometry

of rotation to any point on the fluid trajectory.

ρ = density of fluid in tanks.

D(s) = the perpendicular distance from the virtual center of rotation to tangent at any point on trajectory.

$\alpha(s)$ = angle between r(s) and D(s).

$\gamma(s)$ = angle between r(s) and deck of ship.

A = tank cross-section at s=0 and s=S.

a(s) = tank cross-section at any point s.

g = acceleration of gravity.

3. Euler-LaGrange Equations of Motion.

The two degree of freedom system of ship and tank-fluid was best treated by means of the Euler-LaGrange equations in generalized coordinates. The generalized equation is [Chadwick and Klotter, 1953]

$$\frac{d}{dt}\left[\frac{\partial T}{\partial \dot{q}_i}\right] + \frac{\partial T}{\partial q_i} + \frac{\partial U}{\partial q_i} + \frac{\partial D}{\partial \dot{q}_i} = Q_i \qquad (23)$$

where Q_i is the so-called generalized non-conservative force in the ith equation, and i=1,2. The generalized force is equal to the rate of doing virtual work against the external non-conservative forces when q_i is given a virtual displacement. The parameters of the Euler-LaGrange equation are:

$$T = \sum mv^2/2 \quad = \text{kinetic energy} \qquad (24)$$

$$U = \sum \int f \cdot dx \quad = \text{conservative potential energy (due to gravity)} \qquad (25)$$

$$D = \sum \beta v^2/2 \quad = \text{dissipative energy (due to skin friction, etc.)} \qquad (26)$$

f = conservative force

β = damping parameter

4. Calculation of the Energy Functions.

For each type of energy (kinetic, conservative potential, and dissipative), the contribution of the ship alone and the contribution of the tank-fluid alone was calculated, then the contributions were added to arrive at the total energy. In the regions where the fluid surfaces travel, namely the vertical legs of the U-tube, it was assumed that the tanks were straight, parallel, and of uniform cross-section. This meant that a(s) and D(s) were constants in these regions, thus for any expressions which depended only on the variables a(s) and D(s), the limits of integration were fixed at zero and S, without approximation.

Calculation of kinetic energy:

$$T_{fluid} = \int_{R\tan\theta}^{S+R\tan\theta} \frac{\rho\, a(s)}{2}\left\{\left[\dot{\theta}\frac{R\ A}{a(s)}\right]^2 + \left[\dot{\phi}\, r(s)\right]^2 + 2\left[\dot{\theta}\frac{R\ A}{a(s)}\right]\left[\dot{\phi}\, D(s)\right]\right\} ds \qquad (27)$$

$$T_{ship} = J'_s\, \dot{\phi}^2/2 \qquad (28)$$

$$T_{total} = T_{fluid} + T_{ship} \qquad (29)$$

Calculation of potential energy:

$$U_{fluid} = \left[\rho\ g\ A\ R \int_0^S \frac{a(s)}{A}\ \frac{r(s)}{R}\ \sin\gamma\ ds\right]\left[1 - \cos\phi\right] \qquad (30)$$

$$+\ \rho\ g\ A\ R^2\left[2\tan\theta\sin\phi - \tan^2\theta\ \cos\phi\right]$$

$$U_{ship} = K'_s\, \phi^2/2 \qquad (31)$$

$$U_{total} = U_{fluid} + U_{ship} \qquad (32)$$

Calculation of dissipative energy:

$$D_{fluid} = B'_t \dot{\theta}^2/2 \tag{33}$$

$$D_{ship} = B'_s \dot{\phi}^2/2 \tag{34}$$

$$D_{total} = D_{fluid} + D_{ship} \tag{35}$$

5. The Equations of Motion.

The equations of motion of the ship and tank system come from equation (23), with $q_1 = \phi$, and $q_2 = \theta$. Substituting in equations (27) through (35) and simplifying, and letting $Q_1 = K_s \psi$ and $Q_2 = 0$, the equations of motion are

$$\text{Ship:} \quad (J_s \ddot{\phi} + B_s \dot{\phi} + K_s \phi) + (J_{st} \ddot{\theta} + K_t \theta) = K_s \psi \tag{36}$$

$$\text{Tank:} \quad (J_{st} \ddot{\phi} + K_t \phi) + (J_t \ddot{\theta} + B_t \dot{\theta} + K_t \theta) = 0 \tag{37}$$

Obtaining the system transfer function from the equations of motion was somewhat more complicated than it was for the ship without tanks. First, the equations were divided through by the respective inertia coeficients, and the LaPlace Transform operation was performed:

$$\text{Ship:} \quad \left[s^2 + (B_s/J_s)s + (K_s/J_s)\right]\Phi(s) + \left[(J_{st}/J_s)s^2 + (K_t/J_s)\right]\Theta(s) = (K_s/J_s)\ \Psi(s) \tag{38}$$

$$\text{Tank:} \quad \left[(J_{st}/J_t)s^2 + (K_t/J_t)\right]\Phi(s) + \left[s^2 + (B_t/J_t)s + (K_t/J_t)\right]\Theta(s) = 0 \tag{39}$$

$\Phi(s)$ was isolated by applying Cramer's Rule, and the transfer function was obtained:

$$\Delta = s^4 + \left[\frac{B_sJ_t + J_sB_t}{J_sJ_t - J_{st}^2}\right]s^3 + \left[\frac{K_sJ_t + J_sK_t + B_sB_t - 2K_tJ_{st}}{J_sJ_t - J_{st}^2}\right]s^2$$

$$+ \left[\frac{B_sK_t + K_sB_t}{J_sJ_t - J_{st}^2}\right]s + \left[\frac{K_sK_t - K_t^2}{J_sJ_t - J_{st}^2}\right] \qquad (40)$$

$$\frac{\Phi(s)}{\Psi(s)} = \frac{(K_s/J_s) \quad s^2 + (B_t/J_t)s + (K_t/J_t)}{\Delta} \qquad (41)$$

6. Definitions of Parameters.

The parameters of the equations of motion are given in terms of previously defined parameters and dummy variables as follows:

Ship self-terms:

$$J_s = J'_s + \rho AR^2 \int_0^S \left[a(s)/A\right]\left[r(s)/R\right]^2 ds \qquad (42)$$

$$B_s = B'_s \qquad (43)$$

$$K_s = K'_s + \rho gAR \int_0^S \left[a(s)/A\right]\left[r(s)/R\right] \sin\gamma \; ds \qquad (44)$$

Tank self-terms:

$$J_t = \rho A R^2 \int_0^S A/a(s) \; ds \qquad (45)$$

$$B_t = B'_t \qquad (46)$$

$$K_t = 2\rho g A R^2 \qquad (47)$$

Mutual-coupling terms:

$$J_{st} = \rho A R^2 \int_0^S D(s)/R \; ds \tag{48}$$

$$B_{st} = 0 \tag{49}$$

$$K_{st} = K_t \tag{50}$$

B. ANALYSIS OF THE EQUATIONS OF MOTION.

Having arrived at a set of equations of motion, it was necessary to conduct a brief analysis of the equations to give them physical significance.

1. The Self Terms.

Looking first at the self-terms of the homogeneous equations, it was observed that the ship acts as a simple pendulum if the tank-fluid does not move, and that the tank-fluid acts as a simple pendulum if the ship is prevented from moving. Each of these pendulums has its own natural frequency. These two frequencies are important factors in the stabilization problem.

2. The Mutual Terms.

There is a third critical frequency associated with the ship-tank system, called the secondary resonance frequency, which is due to the mutual-coupling terms. The mathematical significance of the effect became apparent when the ship equation, equation (36), was rewritten with the mutual terms on the right side as follows:

$$J_s \ddot{\phi} + B_s \dot{\phi} + K_s \phi = K_s \psi - (J_{st} \ddot{\theta} + K_t \theta) \tag{51}$$

This equation shows the ship as a simple pendulum, acted upon by a net torque, which is the torque due to waves minus the torque due to tank-fluid motion. If the tank-fluid were made to oscillate at the secondary resonance frequency, the tank-fluid torque would be equal to the wave torque, and thus the ship would be influenced by zero net torque.

C. ANTI-ROLL TANK DESIGN CRITERIA.

The easiest tank geometry to design and build is a right rectangular U-tube, with the vertical legs having constant cross-sectional area, and the horizontal cross-leg having constant but smaller cross-sectional area. Given this tank geometry,

$$a(\text{vertical leg}) = A \tag{52}$$

$$a(\text{horizontal leg}) = a = (A\ R\ \omega_s^2)/(g - h\ \omega_s^2) \tag{53}$$

$$D(\text{vertical leg}) = R \tag{54}$$

$$D(\text{horizontal leg}) = D \tag{55}$$

The length of the trajectory of the fluid in the tank, S, is dependent upon the lever arm of the tank, R, and the height of the fluid column in a vertical leg of the U-tube when $\theta = 0°$, h,

$$S = 2(R + h) \tag{56}$$

For maximum effect, the tank system must span the extreme beam of the ship. Therefore the lever arm of the tank is half of the beam, b, diminished by the width of a vertical leg, w,

$$R = (b-w)/2 \tag{57}$$

The relationships derived from the equations of motion which define the tank dimensions and parameters are:

$$K_t/J_s = \lambda \tag{58}$$

$$K_t/J_t = \omega_t^2$$

$$= \frac{2\ g}{\int_0^S A/a(s)\ ds} \tag{59}$$

$$J_{st}/J_t = X$$

$$X = \int_0^S (D\ a)/(R\ A)\ \ ds$$

$$= 2\ h + 2(D\ a/A) \tag{60}$$

The first ratio, λ, reflects the free surface effect of the tank-fluid, and should be a small number. An acceptable value was found to be 0.05 [Vachanaratana, 1973].

To be effective as a stabilizer, the anti-roll tank system must be "tuned" to the natural frequency of the ship in which it is installed. Therefore,

$$\omega_t^2 = \omega_s^2$$

$$K_t/J_t = K_s/J_s \tag{61}$$

The third ratio, X, is dependent upon the placement of the tanks in the ship relative to the center of rotation. This ratio can be forced to zero (J_{st} forced to zero) by placing the horizontal leg of the U-tube above the center of rotation by the amount

$$-D = h\ A/a \tag{62}$$

There are no hard and fast rules for determining a value

of h, except that the tanks must fit within the hull of the ship, and the fluid in the tanks should not be permitted to either contact the top or completely drain out of the bottom of a vertical leg of the tank at the maximum expected tank-fluid level angle, θ_{max}.

Since J_s depends upon tank geometry, the approximation must be made that $J_s = J_s'$, for purposes of initiating design. Then

$$K_t = \lambda J_s'$$

$$= 2 \rho g A R^2$$

$$= \rho g l w (b-w)^2 / 2 \qquad (63)$$

A reasonable value of tank width should be chosen. Then

$$l = 2 K_t / \rho g w (b-w)^2 \qquad (64)$$

Making the approximation that the natural frequency of the modified ship is the same as the natural frequency of the ship without tanks, and picking a value for h, equation (53) is solved for a, and then equation (62) is tested for feasibility. If the value of D so computed cannot be physically realized, then a realizable value of D must be chosen. This merely means that J_{st} will not be identically zero, and the overall analysis will take more care.

The tank damping parameter is derived by working backwards through the second-order relationships for the hydraulic pendulum, noting that the damping ratio for the tank-fluid is approximately equal to the damping ratio of the (unmodified) ship:

$$\zeta_t = 0.1/\pi \qquad (65)$$

$$B_t = 0.2 \sqrt{K_t J_t} / \pi \qquad (66)$$

Once the tank system parameters have been determined, the modified ship parameters are found. Over the range s=0

to s=h, $r^2 = R^2 + (D+h-s)^2$, and $r \sin\gamma = D+h-s$; over the range s=h to s=h+R, $r^2 = (R+h-s)^2 + D^2$, and $r \sin\gamma = D$.

$$J_s = J'_s + 2\rho A R^2 (I_1 + I_2) \tag{67}$$

$$I_1 = \left[(3R^2+2D^2)h + 2Dh^2 + h^3\right] / \left[3R^2\right] \tag{68}$$

$$I_2 = a(R^2 + 3D^2)/(3AR) \tag{69}$$

$$K_s = K'_s + 2\rho g A R (I_3 + I_4) \tag{70}$$

$$I_3 = -(2D + h^2)/(2R) \tag{71}$$

$$I_4 = Da/A \tag{72}$$

The design steps should then be retraced, removing the approximations. Normally, after two or three iterations of the design procedure, the calculated values converge on the actual values, and a final design has been achieved. The FORTRAN subroutine used in this iterative design procedure is included in Appendix B to this thesis.

D. FREQUENCY RESPONSE OF PASSIVE SHIP-TANK SYSTEM.

The same numerical example used in the frequency response study of a vessel on the ocean was used for the design example and frequency response study of a ship with passive anti-roll tanks to the waveslope spectrum of the ocean:

J'_s = 1289.021 metric tons-meters-sec^2

B'_s = 59.746 metric tons-meters-sec

K'_s = 683.280 metric tons-meters

The following arbitrary choices were made:

$w = 1.0$ meter

$h = 1.3$ meter

$\lambda = 0.05$

$\rho g = 1.025$ metric tons/meter^3 (salt water used)

Then

$$K_t = (0.05)(1289.021)$$
$$= 64.451 \text{ metric tons-meter}$$

$$l = (2)(64.451) / (1.025)(1)(9-1)^2$$
$$= 1.965 \text{ meter}$$

$$A = w\, l$$
$$= 1.965 \text{ meter}^2$$

$$a = \frac{(1.965)(4)(683.28/1289.021)}{9.81 - (1.3)(683.28/1289.021)}$$
$$= 0.457 \text{ meter}^2$$

Testing equation (62) for feasibility,

$$-D = (1.3)(1.965) / 0.457$$
$$= 5.59 \text{ meter}$$

Since this tank placement would probably not be possible in the ship being studied, a more realistic value, still above the center of rotation, was arbitrarily chosen:

$$-D = 1.0 \text{ meter}$$

Then X is non-zero, as indicated.

$$J_{st}/J_t = X$$
$$= 2\left[1.3 - (0.457)(1.965)\right]$$
$$= 2.135$$

To summarize, the tank dimensions found thus far were

$w = 1.0$ meter

$l = 1.965$ meter

$A = 1.965 \text{ meter}^2$

$a = 0.457 \text{ meter}^2$

$R = 4.0 \text{ meter}$

$D = -1.0 \text{ meter}$

Given these tank dimensions, the modified ship parameters became:

$J_s = 1289.021 + 11.155$

$= 1300.176 \text{ metric tons-meters-sec}^2$

$K_s = 683.280 - 3.121$

$= 680.159 \text{ metric tons-meter}$

After four iterations of the design procedure, the following final values were obtained:

$w = 1.0 \text{ meter}$

$l = 1.982 \text{ meter}$

$A = 1.982 \text{ meter}^2$

$a = 0.454 \text{ meter}^2$

$R = 4.0 \text{ meter}$

$D = -1.0 \text{ meter}$

$J_s = 1300.236 \text{ metric tons-meters-sec}^2$

$B_s = 59.746 \text{ metric tons-meters-sec}$

$K_s = 680.185 \text{ metric tons-meters}$

$J_t = 124.276 \text{ metric tons-meters-sec}^2$

$B_t = 5.722 \text{ metric tons-meters-sec}$

$K_t = 65.012 \text{ metric tons-meters}$

$J_{st} = 266.152 \text{ metric tons-meters-sec}^2$

These values were then substituted into the transfer

function, the roots were located, and the frequency response was obtained.

$$\frac{\Phi(s)}{\Psi(s)} = \frac{0.523\,(s^2 + 0.0460\,s + 0.523)}{s^4 + 0.164\,s^3 + 1.485\,s^2 + 0.0857\,s + 0.441} \tag{73}$$

zeroes at $s = -0.0230 \pm j\,0.723$

poles at $s = -0.0135 \pm j\,0.642$

and $s = -0.0684 \pm j\,1.031$

The frequency response is graphically illustrated in Figure 2-2. The DSL/360 program which generated the plot is listed in Appendix B. The negative decibel low frequency response demonstrates the ability of the passive tank system to overcome the natural tendency of the ship to ride normal to the waveslope, making it ride upright with respect to earth reference instead.

Comparing this response with the response of the unmodified ship, given in Figure 1-2, it was observed that the effect of adding passive anti-roll tanks was to place a pair of complex zeroes at the ship's own natural frequency to reduce the resonant peak, from 24 db at 0.7 rad/sec, to two lower peaks of 14 db at 0.6 rad/sec and 10 db at 1.0 rad/sec, and a notch of -11 db at 0.7 rad/sec. In sea state 6, this translates to maximum ship roll angles of 8.5 degrees at 0.6 rad/sec, and 15.8 degrees at 1.0 rad/sec, a considerable improvement over the 34.8 degree maximum roll angle of the unmodified ship.

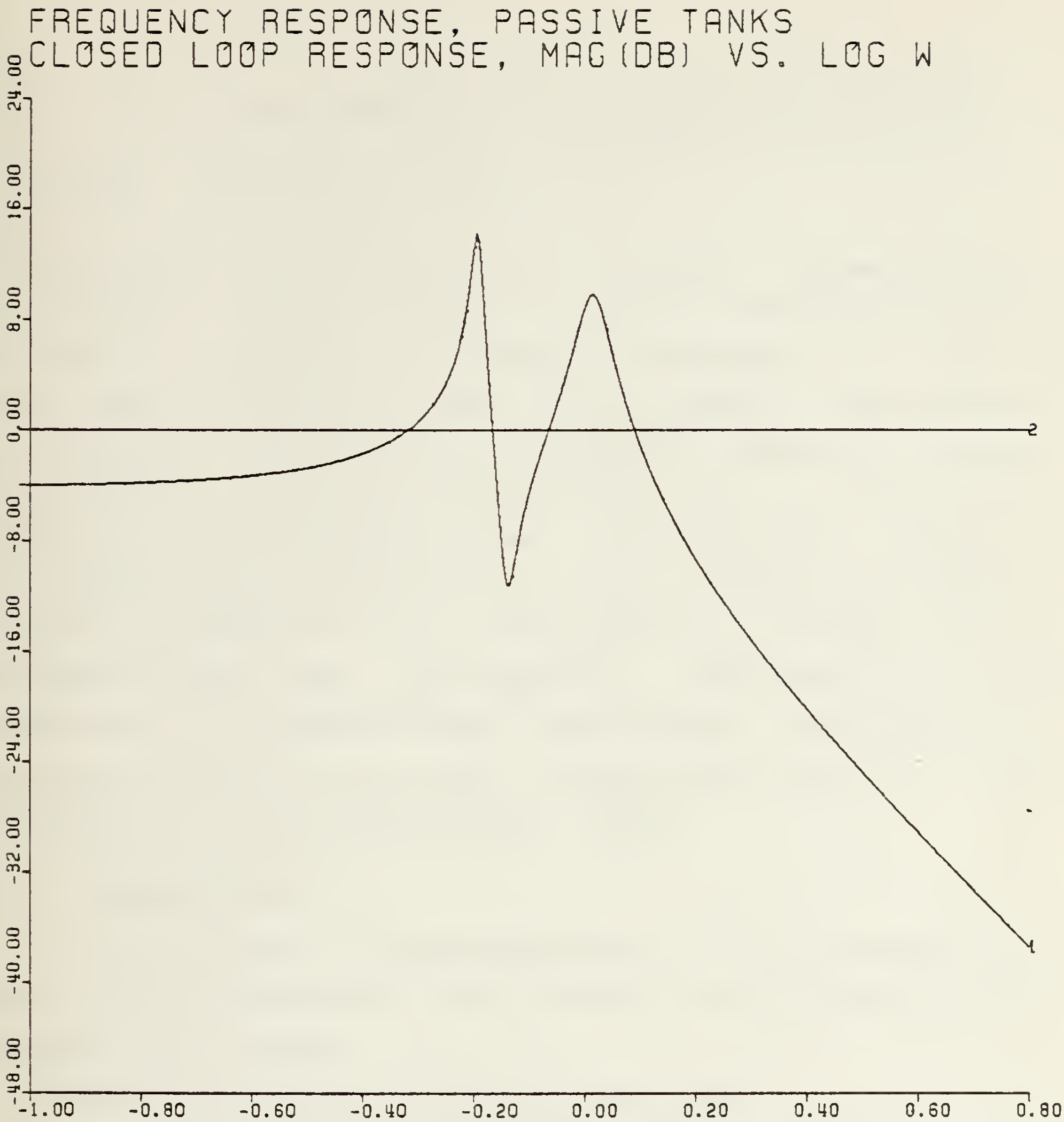


Figure 2.2. Frequency Response of Ship with Passive Tanks

III. SHIP STABILIZATION BY ACTIVATED ANTI-ROLL TANKS

A. MODIFIED EQUATIONS OF MOTION.

Activating an anti-roll tank system, mathematically, only involves adding a forcing function term to the tank equation of motion. Physically, however, it can be accomplished in basically two different ways. These methods are pneumatic and hydraulic. A pneumatic effector operates by alternately creating an overpressure or a partial vacuum condition in the airspace above the tank-fluid surface in the vertical legs of the U-tube, while an hydraulic effector generally consists of an arrangement of pumps in the horizontal cross duct. The effector type chosen for this analysis was a constant speed controllable pitch propellor pump, so chosen for its linear blade pitch angle vs. flow characteristic [Blagoveshchensky, 1962].

1. Nomenclature.

In addition to the nomenclature of the passive tank system, the following are the parameters and variables of an activated tank system:

Dependent variables of the forcing side:

α = instantaneous pump-blade pitch angle (commanded)

Parameters of the forcing variable:

B_p = velocity head-loss coefficient of the pump

K_p = static head coefficient of the pump

Torques produced on the forcing side:

$K_p \alpha - B_p \dot{\theta}$ = Counter-rolling torque due to pump

2. Modification of Parameters.

As indicated by the expression for counter-rolling torque, the addition of a pump to the tank system has the effect of increasing the overall damping coefficient of the tank-fluid. All other parameters of the homogeneous side remain unchanged. The pump should be chosen such that the composite tank damping factor is 0.45 [Vachanaratana, 1973]:

$$B_t = B'_t + B_p \tag{74}$$

$$B_t / J_t = 0.45 \tag{75}$$

$$B_p = 0.45\, J_t - B'_t \tag{76}$$

3. Equations of Motion.

The equations of motion for the activated anti-roll tank system come from equation (23), with $Q_1 = K_s\psi$, $Q_2 = K_p\alpha$, $q_1 = \phi$, and $q_2 = \theta$ [Chadwick and Klotter, 1953].

$$\text{Ship:}\quad (J_s\ddot{\phi} + B_s\dot{\phi} + K_s\phi) + (J_{st}\ddot{\theta} + K_t\theta) = K_s\psi \tag{77}$$

$$\text{Tank:}\quad (J_{st}\ddot{\phi} + K_t\phi) + (J_t\ddot{\theta} + B_t\dot{\theta} + K_t\theta) = K_p\alpha \tag{78}$$

Following the procedure used for the passive ship-tank system, $\Phi(s)$ was found to be

$$\Phi(s) = \Phi_1(s) - \Phi_2(s) \tag{79}$$

$$\Phi_1(s) = \frac{(K_s/J_s)\left[s^2 + (B_t/J_t)s + (K_t/J_t)\right]}{\Delta}\,\Psi(s) \tag{80}$$

$$\Phi_2(s) = \frac{(K_p/J_t)\left[(J_{st}/J_s)s^2 + (K_t/J_s)\right]}{\Delta}\,A(s) \tag{81}$$

Δ = as defined by equation (40)

B. CONTROLLER DESIGN AND FREQUENCY RESPONSE.

The problem of ship roll control is a frequency domain problem: controlling against a (time domain) stochastic

disturbance with a generally known (frequency domain) spectrum. Therefore the classical approach to control was pursued, designing on the basis of feedback compensation. It was assumed that roll angle, rate, and acceleration signals would be available from shipboard sensors (gyros and accelerometers). The controller transfer function was considered to be of the form

$$C(s) = A(s)/\Phi(s)$$

$$= K_2 s^2 + K_1 s + K_0 \tag{82}$$

For purposes of block diagram development and analysis, the following block transfer functions were defined, based upon equations (79), (80), and (81):

$$SS(s) = \Phi_1(s) \,/\, \Psi(s)$$

$$= K_s/J_s \left[s^2 + (B_t/J_t)s + (K_t/J_t)\right] / \Delta$$

$$= \omega_n^2 \left[s^2 + 2\zeta_t \omega_n s + \omega_n^2\right] / \Delta \tag{83}$$

$$ST(s) = \Phi_2(s) \,/\, A(s)$$

$$= K_p/J_t \left[(J_{st}/J_s)s^2 + (K_t/J_s)\right] / \Delta$$

$$= K_p/J_s \left[Xs^2 + \omega_n^2\right] / \Delta \tag{84}$$

$$SST(s) = ST(s) \,/\, SS(s)$$

$$= K_p/K_s \left[Xs^2 + \omega_n^2\right] / \left[s^2 + 2\zeta_t \omega_n s + \omega_n^2\right] \tag{85}$$

$$G(s) = \frac{K_s/J_s \left[s^2 + (B_t/J_t)s + (K_t/J_t)\right]}{\Delta - K_s/J_s \left[s^2 + (B_t/J_t)s + (K_t/J_t)\right]}$$

$$= \frac{\omega_n^2 \left[s^2 + 2\zeta_t \omega_n s + \omega_n^2\right]}{\Delta - \omega_n^2 \left[s^2 + 2\zeta_t \omega_n s + \omega_n^2\right]} \tag{86}$$

$$H(s) = SST(s)\,C(s)$$

$$= \frac{1/K_s\left[XC_2 s^4 + XC_1 s^3 + (XC_0 + \omega_n^2 C_2)s^2 + \omega_n^2 C_1 s + \omega_n^2 C_0\right]}{s^2 + 2\zeta_t \omega_n s + \omega_n^2} \qquad (87)$$

Where

$$C_2 = K_p K_2$$

$$C_1 = K_p K_1$$

$$C_0 = K_p K_0$$

The "open loop" transfer function, then, was

$$F(s) = G(s) \,/\, \left[1 + G(s)H(s)\right]$$

$$= \frac{(K_s/J_s)\left[s^2 + (B_t/J_t)s + (K_t/J_t)\right]}{\Delta(\text{open})} \qquad (88)$$

$$\Delta(\text{open}) = \Delta - (K_s/J_s)\left[s^2 + (B_t/J_t)s + (K_t/J_t)\right] +$$

$$+ (1/J_s)\left[XC_2 s^4 + X\,C_1 s^3 + (XC_0 + (K_t/J_t)C_2)s^2\right.$$

$$\left. + (K_t/J_t)C_1 s + (K_t/J_t)C_0\right] \qquad (89)$$

The overall, or "closed loop" response became

$$T(s) = \Phi(s) \,/\, \Psi(s)$$

$$= F(s) \,/\, \left[1 + F(s)\right]$$

$$= \frac{(K_s/J_s)\left[s^2 + (B_t/J_t)s + (K_t/J_t)\right]}{\Delta(\text{closed})} \qquad (90)$$

$$\Delta(\text{closed}) = \Delta + (1/J_s)\left[XC_2 s^4 + XC_1 s^3 + (XC_0 + (K_t/J_t)C_2)s^2\right.$$

$$\left. + (K_t/J_t)C_1 s + (K_t/J_t)C_0\right] \qquad (91)$$

The Characteristic Equation of the closed loop response, Δ(closed), was

$$s^4 + A_3 s^3 + A_2 s^2 + A_1 s + A_0 = 0 \tag{92}$$

$$A_3 = \left[\frac{J_s J_t}{J_s J_t + J_{st} C_2}\right] \left[\frac{B_s J_t + J_s B_t}{J_s J_t + J_{st}^2} + \frac{J_{st} C_1}{J_s J_t}\right] \tag{93}$$

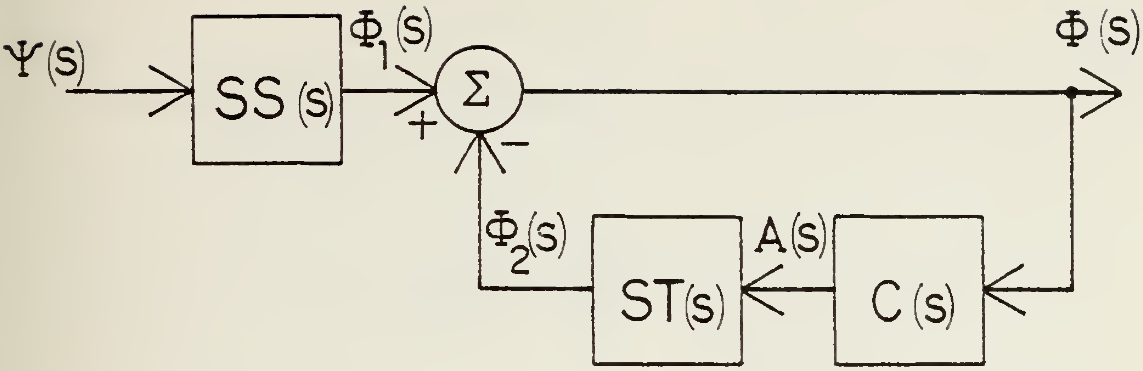
$$A_2 = \left[\frac{J_s J_t}{J_s J_t + J_{st} C_2} \quad \frac{K_s J_t + J_s K_t + B_s B_t - 2K_t J_{st}}{J_s J_t + J_{st}^2} + \frac{J_{st} C_0 + K_t C_2}{J_s J_t}\right] \tag{94}$$

$$A_1 = \left[\frac{J_s J_t}{J_s J_t + J_{st} C_2}\right] \left[\frac{B_s K_t + K_s B_t}{J_s J_t - J_{st}^2} + \frac{K_t C_1}{J_s J_t}\right] \tag{95}$$

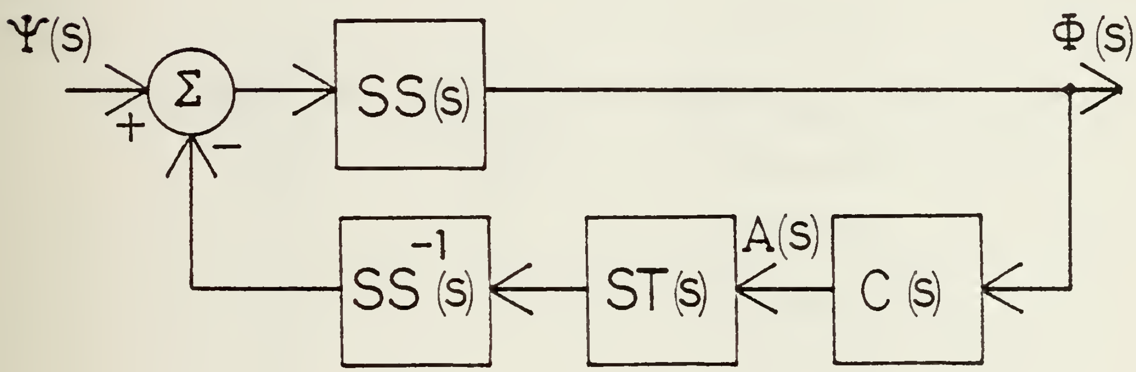
$$A_0 = \left[\frac{J_s J_t}{J_s J_t + J_{st} C_2}\right] \left[\frac{K_s K_t - K_t^2}{J_s J_t - J_{st}^2} + \frac{K_t C_0}{J_s J_t}\right] \tag{96}$$

The block diagram development is illustrated in Figure 3-1. The first design approach attempted was the modern control theory state variable feedback method. The difficulty with this method was the inability to define a time-domain objective or cost functional.

The next design approach attempted was with the Computer Aided Design of Optimal Compensators (CADOC) program [Anastassakis, 1977]. This method consisted of selecting a frequency-domain cost functional and a desired frequency response curve, and used the function minimization procedure of M.J. Box. There were two difficulties encountered in applying this program to the problem of ship anti-roll

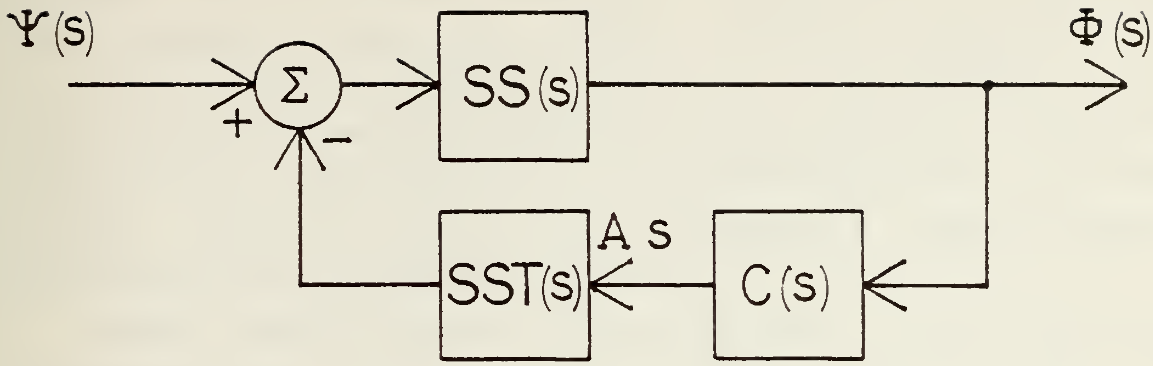


(a) Direct implementation of equations (79) and (82).

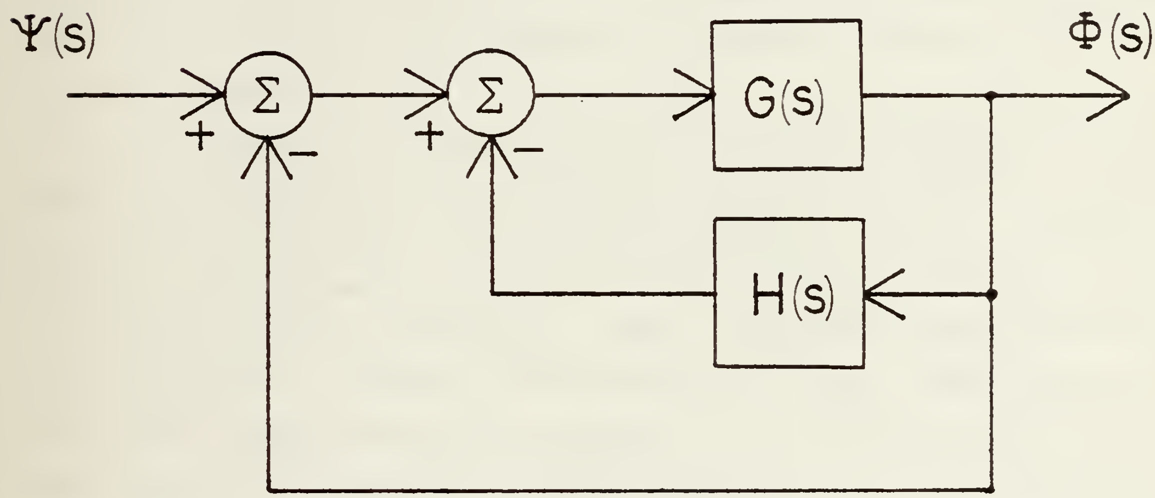


(b) First state of block diagram reduction.

Figure 3.1. Block Diagram Development for Activated Ship/Tank System



(c) Final block diagram.



(d) Equivalent block diagram for servo theory analysis.

Figure 3.1. Block Diagram Development for Activated Ship/Tank System

control. The first problem was that a portion of the feedback path in the physical system was fixed by architecture, while the CADOC program required all coefficients in the feedback path to be adjustable. The second problem involved selecting an attainable frequency response for CADOC input. It was concluded that the CADOC program was best suited for optimizing a close design, rather than for initiating a design.

Another design approach attempted was in the root locus parameter space. Two separate procedures exist here, and both were attempted. The first procedure was the standard root locus approach, fixing two of the three adjustable gains and varying the third. This method amounted to an exhaustive search technique, attempting to locate the best combination of the three gains to define system roots with adequate damping.

The second root locus parameter space procedure involved using a parameter plane computer program, PARAM A, which exists at the Naval Postgraduate School and is available from Dr. G.J. Thaler, but has not been documented in any literature to his knowledge. This program varies two gains which appear in linear combinations in the coefficients of the system characteristic equation, for a fixed value of the third gain. The output of this program is a two-dimensional parameter plane showing families of curves of constant damping ratio and constant natural frequency as functions of the two gains.

The problems associated with both root locus parameter space procedures were the exhaustive search nature of a three-variable system, and the exclusions of the system closed-loop zeroes from the analysis. This was noted to be a problem when the gains found by these procedures were used in a frequency response study of the activated ship/tank system, and large resonant peaks were observed.

After attempting all of the above methods and abandoning them for the reasons stated, an exhaustive search of closed-loop frequency response curves was conducted, using the program listed in Appendix C. An acceptable design was accomplished when C = 110, C = -150, and C = -80. This corresponded to placing the compensator roots at s = -2.4388 and s = +0.5638, which improved the overall system damping and provided the frequency response illustrated in Figure 3-2. This response was characterized by two low resonant peaks, one at 0.582 rad/sec of 2.85 db (linear gain 1.39), and one at 1.38 rad/sec of 1.96 db (linear gain 1.25). The minimum response between the two resonances occured at 0.861 rad/sec, and was -4.75 db (linear gain 0.58).

In sea state 6, the maximum waveslopes at the resonant frequencies are 1.44 degrees at 0.582 rad/sec, and 8.11 degrees at 1.38 rad/sec, and the maximum waveslope at 0.861 rad/sec is 3.15 degrees. Using this controller design, the maximum roll angles that the ship should experience in sea state 6 would be 2.0 degrees at 0.582 rad/sec, 1.83 degrees at 0.861 rad/sec, and 10.1 degrees at 1.38 rad/sec. The wave height power spectral density at those frequencies, according to the spectra of Pierson and Moskowitz, are 205 ft^2/Hz at 0.582 rad/sec (0.093 Hz), 80 ft^2/Hz at 0.861 rad/sec (0.137 Hz), and 12 ft^2/Hz at 1.38 rad/sec (0.219 Hz). Therefore it was concluded that even though the pure numbers indicated a poor roll control at the higher frequency resonant peak, the waves at that frequency lack sufficient power to cause the ship to attain the maximum theoretical roll angle.

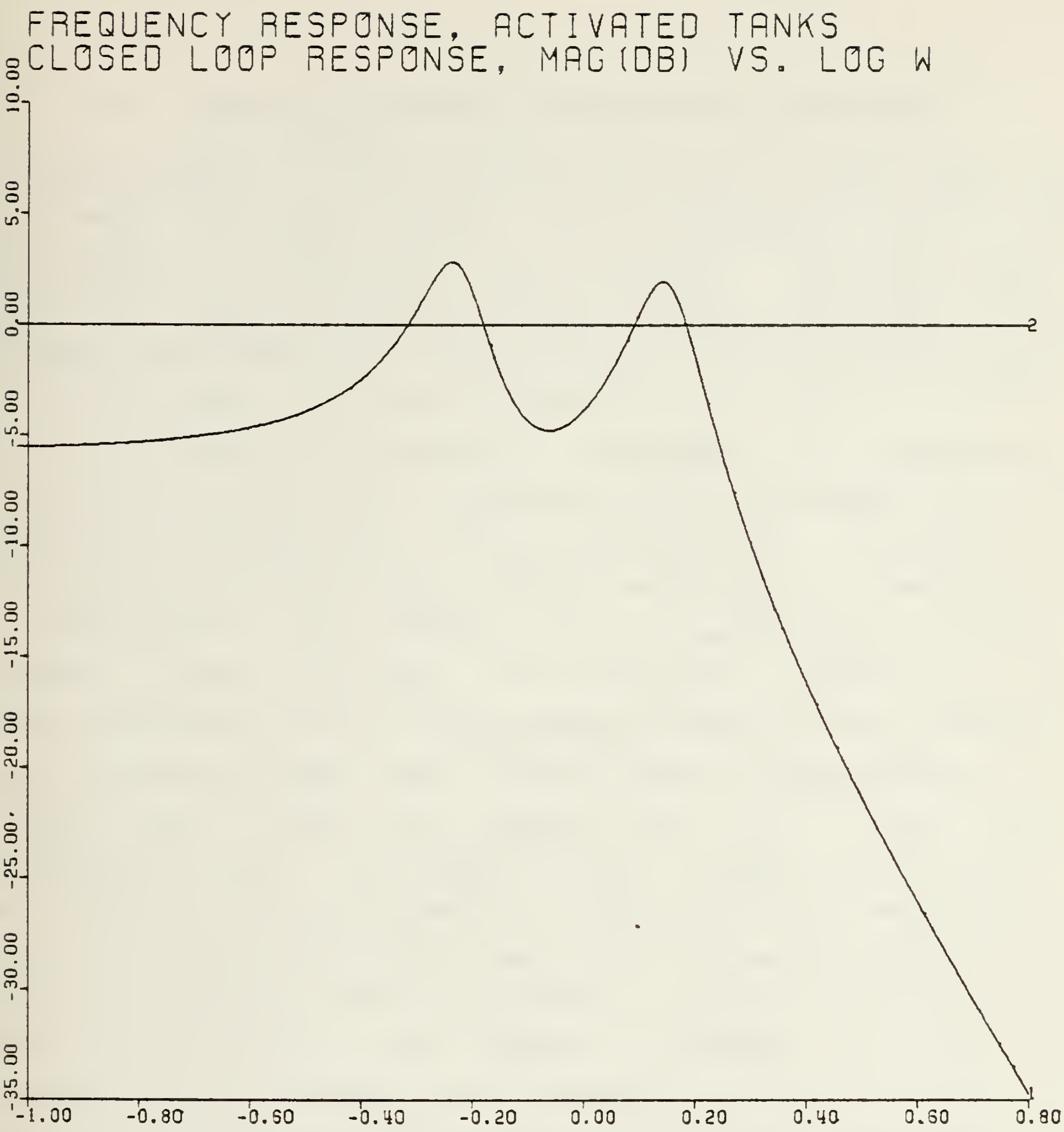


Figure 3-2. Frequency Response of Activated Ship/Tank System, Proportional Plus Rate Plus Acceleration Control

IV. EFFECTS OF VESSEL DISPLACEMENT VARIATIONS

The analysis of ship motions and roll control thus far had been conducted under the assumption that the displacement of the vessel was constant. Over the short term, the assumption may be true enough, but for a ship underway, consuming fuel and stores, the assumption cannot hold true beyond a few days at best.

As a ship's displacement decreases, the underwater surface area of the hull decreases also, in keeping with the Principle of Archimedes. This in turn leads to a lowering of the Center of Buoyancy, which results in a lowering of the Metacenter. The effect on the metacentric height, however, is not so clear, as the exact location of the Center of Gravity in the ship depends more on the placement of the onboard weight than on the total displacement. In general, fuel tanks are located low in a ship; thus consumption of fuel is removal of low weight, which shifts the Center of Gravity upward, decreasing the metacentric height. It should be noted here that ballasting empty fuel tanks with sea water has the effect of restoring the ship as much as possible to the original condition of statical stability which existed at the beginning of the voyage, and extends the time frame of applicability of the assumption of constant displacement.

Varying the ship's displacement therefore varies the natural frequency of the ship, and effectively de-tunes the anti-roll tank system. The only physical adjustment that can be made to a passive tank installation underway is to vary the quantity of water in the U-tube. This has the effect of changing the limits of integration of the design equations, and therefore changes the parameters of the tank.

Additional adjustments are available on the activated tank system. These are the gain of the pump, which is a function of the pump rotational speed, and the feedback gains from the sensors to the pump controls. Thus, if a method exists for determining on-line the various ship parameters as they change, the controller for the activated tank system can be made adaptive. Various schemes may be considered, from Kalman filter type parameter estimators to FFT devices, and this field is left as a topic for further possible research.

V. CONCLUSIONS

In studying ship roll control, it was necessary to become familiar with the nature of ocean waves. The waveslope spectrum of the ocean which was generated, when used in conjunction with the Pierson-Moskowitz wave power spectra, gave much insight into the problem. It was learned that the sea surface does not and can not assume the slopes that an observer on a ship often estimates.

A typical ship on the ocean magnifies the waveslopes near the resonant frequency by a factor of about 15. By installing passive anti-roll tanks, this waveslope magnification factor was split into two resonant peaks, one with a factor of 5, and the other with a factor of 3. Activating the tank system further decreased the waveslope magnification factors to 1.4 and 1.3, respectively.

Several assumptions have been made over the years in the study of roll control by U-tube tank systems, and some of them were sources of problems in the design of physical systems. The first assumption was that a ship has constant parameters: displacement, metacentric height, and natural period. This assumption holds true only if the ship does not consume any fuel or stores, or if these weights are replaced with appropriate quantities of suitably placed ballast.

Another assumption was that the tanks could be placed in the ship in such fashion as to drive the mutual-coupling inertia coefficient to zero. This assumption had two limitations. The first pitfall was the direct connection between this assumption and the assumption of constant ship parameters. The second pitfall was that the tanks had to be placed in an impossible location on the ship to accomplish the objective of the assumption.

Roll control by pump activated tanks is perhaps the best form of roll control for slow moving vessels, or for vessels conducting operations either at slow speed or stopped, where activated fin stabilizers lose their effectiveness. It should be noted that a pump activated tank system can be used to cause rolling, as an aid for breaking out of ice. It can also be used to correct list, including damage-caused list, as a safer alternative to counter-flooding. Therefore the pump activated U-tube tank system is the most versatile form of roll control that a multi-purpose vessel can have installed.

APPENDIX A

Program Listing for Frequency Response of Unmodified Ship, with FORTRAN Subroutine PARAM

The Frequency Response program was written in Digital Simulation Language (DSL/360) to be run on the IBM System/360 machine at the W. R. Church Computer Center, Naval Postgraduate School. To use the program, a deck of plot identification cards must be appended following the FORTRAN Subroutine, in accordance with the DSL/360 user's manual.

The labels and variable names used in the program are defined in the Glossary for Frequency Response Programs, Appendix D of this thesis.

```
// EXEC DSL
//DSL.INPUT DD *
INTGER NPLOT
CONST NPLOT=1
TITLE FREQUENCY RESPONSE OF SHIP WITHOUT TANKS
D      COMPLEX S,D,N,G,H,Q
D      COMMON/CAREA/S,D,N,G,H,Q
INITIAL
       CALL PARAM(JS,BS,KS)
       PHI180=-180.0
       DBZERO=C.C
DYNAMIC
       LOGW=RAMP(0.)-1.
       W=10.**LOGW
       S=CMPLX(0.,W)
       N = KS/JS
       Q = S**2 + (BS/JS)*S + (KS/JS)
       D = Q - N
       G=N/D
       H=G/(1+G)
       RE=REAL(G)
       IM=AIMAG(G)
       REH=REAL(H)
       IMH=AIMAG(H)
       PHI=57.3*ATAN2(IM,RE)
       IF(PHI.GT.0.)PHI=PHI-360.
       MAG=CABS(G)
       MAGH=CABS(H)
       MDB=20.*ALOG10(MAG)
       MDBH=20.*ALOG10(MAGH)
       GREAL=LIMIT(-15.0,10.0,RE)
       GIMAG=LIMIT(-15.0,10.0,IM)
       PHIDMY=-180.0+100.0*RAMP(0.0)
       MDBDMY=-35.0+25.0*RAMP(0.0)
SAMPLE
       CALL DRWG(1,1,LOGW,MDB)
       CALL DRWG(1,2,LOGW,DBZERO)
       CALL DRWG(2,1,LOGW,PHI)
       CALL DRWG(2,2,LOGW,PHI180)
       CALL DRWG(3,1,PHI,MDB)
       CALL DRWG(3,2,PHI180,MDBDMY)
       CALL DRWG(3,3,PHIDMY,DBZERO)
       CALL DRWG(4,1,GREAL,GIMAG)
       CALL DRWG(4,2,MDBDMY,DBZERO)
       CALL DRWG(4,3,DBZERO,MDBDMY)
```

```
      CALL  DRWG(5,1,LOGW,MDBH)
      CALL  DRWG(5,2,LOGW,DBZERO)
TERMINAL
      CALL  ENDRW(NPLOT)
CONTRL FINTIM=1.8,DELT=0.005,DELS=0.005
PRINT .025,W,PHI,RE,IM,MDB,MAG,MDBH,MAGH
END
STOP
```

```
FORTRAN
      SUBROUTINE PARAM(JS,BS,KS)
      REAL KRAD,JS,KS,LEN
      LEN = 49.1
      BEAM=  9.0
      DISP=936.0
      GM  =  0.73
      TPHI=  8.63
      GRAV=  9.81
      PI  =  3.141593
C...  CALCULATION OF UNMODIFIED SHIP PARAMETERS
      KRAD= TPHI * SQRT(GRAV*GM)/(2.0*PI)
      JS  = (CISP/GRAV) * KRAD**2
      BS  = 0.1 * TPHI * DISP * GM / PI**2
      KS  = DISP * GM
      WRITE(6,100) JS,BS,KS
100   FORMAT('1UNMODIFIED SHIP PARAMETERS ARE'/
     1       '0   JS = ',1F12.5/
     2       '0   BS = ',1F12.5/
     3       'C   KS = ',1F12.5//)
      RETURN
      END
```

APPENDIX B

Program Listing for Frequency Response of Ship with Passive Tanks, with FORTRAN Subroutine CALC

The Frequency Response program was written in Digital Simulation Language (DSL/360) to be run on the IBM System/360 machine at the W. R. Church Computer Center, Naval Postgraduate School. To use the program, a deck of plot identification cards must be appended following the FORTRAN Subroutine, in accordance with the DSL/360 user's manual.

The labels and variable names used in the program are defined in the Glossary for Frequency Response Programs, Appendix D of this thesis.

```
// EXEC DSL
//DSL.INPUT DD *
INTGER NPLOT
CONST  NPLOT=1
TITLE  FREQUENCY RESPONSE OF SHIP-TANK SYSTEM (PASSIVE)
D      COMPLEX S,D,N,G,H,Q
D      COMMON/CAREA/S,D,N,G,H,Q
INITIAL
       CALL CALC(JS,BS,KS,JT,BT,KT,JST)
       PHI180=-180.0
       DBZERO=0.0
       DENOM = JS*JT - JST**2
       A0 = (KS*KT - KT**2)/DENCM
       A1 = (BS*KT + KS*BT)/DENCM
       A2 = (KS*JT + JS*KT + BS*BT - 2.0*KT*JST)/DENOM
       A3 = (BS*JT + JS*BT)/DENOM
DYNAMIC
       LOGW=RAMP(0.)-1.
       W=10.**LCGW
       S=CMPLX(0.,W)
       N = KS/JS*(S**2 + (BT/JT)*S + (KT/JT))
       Q = S**4 + A3*S**3 + A2*S**2 + A1*S + A0
       D = Q - N
       G=N/D
       H=G/(1+G)
       RE=REAL(G)
       IM=AIMAG(G)
       REH=REAL(H)
       IMH=AIMAG(H)
       PHI=57.3*ATAN2(IM,RE)
       IF(PHI.GT.0.)PHI=PHI-360.
       MAG=CABS(G)
       MAGH=CABS(H)
       MDB=20.*ALOG10(MAG)
       MDBH=20.*ALOG10(MAGH)
       GREAL=LIMIT(-15.0,10.0,RE)
       GIMAG=LIMIT(-15.0,10.0,IM)
       PHIDMY=-180.0+100.0*RAMP(0.0)
       MDBDMY=-35.0+25.0*RAMP(0.0)
SAMPLE
       CALL DRWG(1,1,LOGW,MDB)
       CALL DRWG(1,2,LOGW,DBZERO)
       CALL DRWG(2,1,LOGW,PHI)
       CALL DRWG(2,2,LOGW,PHI180)
       CALL DRWG(3,1,PHI,MDB)
```

```
      CALL DRWG(3,2,PHI180,MDBDMY)
      CALL DRWG(3,3,PHIDMY,DBZERO)
      CALL DRWG(4,1,GREAL,GIMAG)
      CALL DRWG(4,2,MDBDMY,DBZERO)
      CALL DRWG(4,3,DBZERO,MDBDMY)
      CALL DRWG(5,1,LOGW,MDBH)
      CALL DRWG(5,2,LOGW,DBZERO)
TERMINAL
      CALL ENDRW(NPLOT)
CONTRL FINTIM=1.8,DELT=0.005,DELS=0.005
PRINT .025,W,PHI,RE,IM,MDB,MAG,MDBH,MAGH
END
STOP
```

```
FORTRAN
      SUBROUTINE CALC(JS,BS,KS,JT,BT,KT,JST)
      REAL LEN,L,I1,I2,I3,I4,LMDA,JS,KS,JS1,KS1,JT,KT,JST,KRAD
      LEN = 49.1
      BEAM=  9.0
      DISP=936.0
      GM  =  0.73
      TPHI=  8.63
      GRAV=  9.81
      RHOG=  1.025
      PI  =  3.141593
      WIDE=  1.0
      LMDA=  0.05
      D   =- 1.0
      H   =  1.3
      N   =  0
C...  CALCULATION OF UNMODIFIED SHIP PARAMETERS
      KRAD= TPHI * SQRT(GRAV*GM)/(2.0*PI)
      JS1 = (DISP/GRAV) * KRAD**2
      BS  = 0.1 * TPHI * DISP * GM / PI**2
      KS1 = DISP * GM
      JS  = JS1
      KS  = KS1
      WRITE(6,100) JS,BS,KS
100   FORMAT('1UNMODIFIED SHIP PARAMETERS ARE'/
     1       'C   JS = ',1F12.5/
     2       '0   BS = ',1F12.5/
     3       '0   KS = ',1F12.5//)
C...  CALCULATION OF TANK PARAMETERS
      RHO = RHOG/GRAV
      R   = (BEAM - WIDE)/2.0
10    WN2 = KS/JS
      KT  = LMDA * JS
      L   = KT / (2.0 * RHOG * WIDE * R**2)
      A   = L * WIDE
      AS  = (A * R * WN2)/(GRAV - H * WN2)
      DCAL= -1.0*(H * A/AS)
      DABS= ABS(DCAL)
      IF(DABS.LT.2.5) D=DCAL
      X   = 2.0 * (H + D*AS/A)
      JT  = KT/WN2
      JST = JT * X
      BT  = 0.2*SQRT(KT*JT)/PI
C...  CALCULATION OF MODIFIED SHIP PARAMETERS
      I1  = ((3.0*R**2+2.0*D**2)*H+2.0*D*H**2+H**3)/(3.0*R**2)
```

```
      I2   = AS*(R**2+3.0*D**2)/(3.0*A*R)
      I3   = -(2.0*D+H**2)/(2.0*R)
      I4   = AS*D/A
      TJ   = JS1+2.0*RHO*A*R**2*(I1+I2)
      TK   = KS1+2.0*RHOG*A*R*(I3+I4)
      E1   = ABS(JS-TJ)
      E2   = ABS(KS-TK)
      JS   = TJ
      KS   = TK
      N    = N+1
      IF(N.GT.500) GO TO 20
      IF (E1.GT.0.0001) GO TO 10
      IF (E2.GT.0.0001) GO TO 10
20    WRITE(6,110)DCAL
110   FORMAT('0THE CALCULATED VALUE OF D TO FORCE JST TO ZERO WAS'/
     4        3X,1F12.5//)
      WRITE(6,120)N
120   FORMAT('0AFTER ',I3,' ITERATIONS,')
      WRITE(6,130)L,WIDE,A,AS,R,D,H,X
130   FORMAT('0THE TANK DIMENSIONS AND PARAMETERS ARE'/
     5        '0VERTICAL LEG LENGTH      L  = ',1F12.5/
     6        '0VERTICAL LEG WIDTH       W  = ',1F12.5/
     7        '0VERTICAL LEG AREA        A  = ',1F12.5/
     8        '0HORIZCNTAL LEG AREA      AS = ',1F12.5/
     9        '0LEVER ARM OF TANK        R  = ',1F12.5/
     A        '0REF. CENTER OF ROTATICN  D  = ',1F12.5/
     B        '0HEIGHT OF WATER COLUMN   H  = ',1F12.5/
     C        '0THE RATIO                X  = ',1F12.5//)
      WRITE(6,140)JS,BS,KS,JT,BT,KT,JST
140   FORMAT('0THE MODIFIED SHIP AND TANK SYSTEM PARAMETERS ARE'/
     D        '0      JS  = ',1F12.5/
     E        'C      BS  = ',1F12.5/
     F        '0      KS  = ',1F12.5/
     G        '0      JT  = ',1F12.5/
     H        '0      BT  = ',1F12.5/
     I        '0      KT  = ',1F12.5/
     J        '0      JST = ',1F12.5//)
      RETURN
      END
```

APPENDIX C

Program Listing for Frequency Response of Activated Ship/Tank System, with FORTRAN Subroutine CALCA

The Frequency Response program was written in Digital Simulation Language (DSL/360) to be run on the IBM System/360 machine at the W. R. Church Computer Center, Naval Postgraduate School. To use the program, a deck of plot identification cards must be appended following the FORTRAN Subroutine, in accordance with the DSL/360 user's manual.

The labels and variable names used in the program are defined in the Glossary for Frequency Response Programs, Appendix D of this thesis.

```
// EXEC DSL
//DSL.INPUT CD *
INTGER NPLOT
CONST NPLOT=1
TITLE FREQUENCY RESPONSE OF ACTIVATED SHIP-TANK SYSTEM
TITLE PROPORTIONAL PLUS RATE PLUS ACCELERATION CONTROL
D      COMPLEX S,D,N,G,H,Q,SST,COMP,OLOOP,CLOOP
D      COMMON/CAREA/S,D,N,G,H,Q,SST,COMP,OLOOP,CLOOP
PARAM CO =  110.0
PARAM C1 = -150.0
PARAM C2 = - 80.0
INITIAL
       CALL CALCA(JS,BS,KS,JT,BT,KT,JST)
       PHI180=-180.0
       DBZERO=0.0
       DENOM= JS*JT - JST**2
       A0   = (KS*KT - KT**2)/DENOM
       A1   = (BS*KT + KS*BT)/DENOM
       A2   = (KS*JT + JS*KT + BS*BT - 2.0*KT*JST)/DENOM
       A3   = (BS*JT + JS*BT)/DENOM
DYNAMIC
       LOGW=RAMP(0.)-1.
       W=10.**LOGW
       S=CMPLX(0.,W)
       N=(KS/JS)*(S**2 + (BT/JT)*S + (KT/JT))
       Q=S**4 + A3*S**3 + A2*S**2 + A1*S + A0
       D=Q-N
       G=N/D
       COMP=C2*S**2 + C1*S + CO
       SST=((JST/JT)*S**2 + (KT/JT))/(KS*N)
       H=SST*COMP
       OLOOP=G/(1+G*H)
       CLOOP=CLOOP/(1+OLOOP)
       RE=REAL(OLOOP)
       IM=AIMAG(CLOOP)
       REH=REAL(CLOOP)
       IMH=AIMAG(CLOOP)
       PHI=57.3*ATAN2(IM,RE)
       IF(PHI.GT.0.)PHI=PHI-360.
       MAG=CABS(OLOOP)
       MDB=20.*ALOG10(MAG)
       MAGH=CABS(CLOOP)
       MDBH=20.*ALOG10(MAGH)
       GREAL=LIMIT(-15.0,10.0,RE)
       GIMAG=LIMIT(-15.0,10.0,IM)
```

```
      PHIDMY=-180.0+100.0*RAMP(0.0)
      MDBDMY=-35.0+25.0*RAMP(0.0)
SAMPLE
      CALL  DRWG(1,1,LOGW,MDB)
      CALL  DRWG(1,2,LOGW,DBZERO)
      CALL  DRWG(2,1,LOGW,PHI)
      CALL  DRWG(2,2,LOGW,PHI180)
      CALL  DRWG(3,1,PHI,MDB)
      CALL  DRWG(3,2,PHI180,MDBDMY)
      CALL  DRWG(3,3,PHIDMY,DBZERO)
      CALL  DRWG(4,1,GREAL,GIMAG)
      CALL  DRWG(4,2,MDBDMY,DBZERO)
      CALL  DRWG(4,3,DBZERO,MDBDMY)
      CALL  DRWG(5,1,LOGW,MDBH)
      CALL  DRWG(5,2,LOGW,DBZERO)
TERMINAL
      CALL  ENDRW(NPLOT)
CONTRL FINTIM=1.8,DELT=0.005,DELS=0.005
PRINT .005,W,PHI,RE,IM,MDB,MAG,MDBH,MAGH
END
STOP
```

```
FORTRAN
      SUBROUTINE CALCA(JS,BS,KS,JT,BT,KT,JST)
      REAL LEN,L,I1,I2,I3,I4,LMDA,JS,KS,JS1,KS1,JT,KT,JST,KRAD
      LEN = 49.1
      BEAM=  9.0
      DISP=936.0
      GM  =  0.73
      TPHI=  8.63
      GRAV=  9.81
      RHOG=  1.025
      PI  =  3.141593
      WIDE=  1.0
      LMDA=  0.05
      D   =- 1.0
      H   =  1.3
      DAMP=  0.45
      N   =  C
C...  CALCULATION OF UNMODIFIED SHIP PARAMETERS
      KRAD= TPHI * SQRT(GRAV*GM)/(2.0*PI)
      JS1 = (DISP/GRAV) * KRAD**2
      BS  = 0.1 * TPHI * DISP * GM / PI**2
      KS1 = DISP * GM
      JS  = JS1
      KS  = KS1
      WRITE(6,100) JS,BS,KS
100   FORMAT('1UNMODIFIED SHIP PARAMETERS ARE'/
     1      '0   JS = ',1F12.5/
     2      'C   BS = ',1F12.5/
     3      '0   KS = ',1F12.5//)
C...  CALCULATION OF TANK PARAMETERS
      RHO = RHOG/GRAV
      R   = (BEAM - WIDE)/2.0
10    WN2 = KS/JS
      KT  = LMCA * JS
      L   = KT / (2.0 * RHOG * WIDE * R**2)
      A   = L * WIDE
      AS  = (A * R * WN2)/(GRAV - H * WN2)
      DCAL= -1.0*(H * A/AS)
      DABS= ABS(DCAL)
      IF(DABS.LT.2.5) D=DCAL
      X   = 2.0 * (H + D*AS/A)
      JT  = KT/WN2
      JST = JT * X
      BT1 =  0.2*SQRT(KT*JT)/PI
      BP  = CAMP * JT - BT1
```

```
      BT   = BT1 + BP
C...  CALCULATION OF MODIFIED SHIP PARAMETERS
      I1   = ((3.0*R**2+2.0*D**2)*H+2.0*D*H**2+H**3)/(3.0*R**2)
      I2   = AS*(R**2+3.0*D**2)/(3.0*A*R)
      I3   = -(2.0*D+H**2)/(2.0*R)
      I4   = AS*D/A
      TJ   = JS1+2.0*RHO*A*R**2*(I1+I2)
      TK   = KS1+2.0*RHOG*A*R*(I3+I4)
      E1   = ABS(JS-TJ)
      E2   = ABS(KS-TK)
      JS   = TJ
      KS   = TK
      N    = N+1
      IF (N.GT.99)GO TO 20
      IF (E1.GT.0.00001)  GO TO 10
      IF (E2.GT.0.00001)  GO TO 10
20    WRITE(6,110)DCAL
110   FORMAT('OTHE CALCULATED VALUE OF D TO FORCE JST TO ZERO WAS'/
     4       3X,1F12.5//)
      WRITE(6,120)N
120   FORMAT('OAFTER ',I3,' ITERATIONS,')
      WRITE(6,130)L,WIDE,A,AS,R,D,H,X
130   FORMAT('OTHE TANK DIMENSIONS AND PARAMETERS ARE'/
     5       'OVERTICAL LEG LENGTH      L  = ',1F12.5/
     6       'OVERTICAL LEG WIDTH       W  = ',1F12.5/
     7       'OVERTICAL LEG AREA        A  = ',1F12.5/
     8       'OHORIZONTAL LEG AREA      AS = ',1F12.5/
     9       'OLEVER ARM OF TANK        R  = ',1F12.5/
     A       'OREF. CENTER OF ROTATION  D  = ',1F12.5/
     B       'OFEIGHT OF WATER COLUMN   H  = ',1F12.5/
     C       'OTHE RATIO                X  = ',1F12.5//)
      WRITE(6,140)DAMP,BP
140   FORMAT('OTO YIELD A TANK DAMPING FACTOR OF ',1F3.2/
     D       'OTHE PUMP MUST HAVE A BP = ',1F12.5//)
      WRITE(6,150)JS,BS,KS,JT,BT,KT,JST
150   FORMAT('OTHE MODIFIED SHIP AND TANK SYSTEM PARAMETERS ARE'/
     E       '0      JS  = ',1F12.5/
     F       '0      BS  = ',1F12.5/
     G       '0      KS  = ',1F12.5/
     H       '0      JT  = ',1F12.5/
     I       '0      BT  = ',1F12.5/
     J       '0      KT  = ',1F12.5/
     K       '0      JST = ',1F12.5//)
      RETURN
      END
```

APPENDIX D

Glossary for Frequency Response Programs

MAIN

SYMBOL	MEANING
A0,A1,A2,A3	Coefficients of characteristic equation
BS	Damping coefficient of ship
BT	Overall damping coefficient of tank-fluid
CLOOP	Closed-loop transfer function
COMP	Compensator (controller) transfer function
D	Denominator polynomial of open-loop transfer function
G	Open-loop transfer function
GIMAG	Imaginary part of open-loop transfer function
GREAL	Real part of open-loop transfer function
H	Closed-loop transfer function
IM	Imaginary part of open-loop transfer function
IMH	Imaginary part of open-loop transfer function
JS	Roll inertia coefficient of ship
JST	Mutual-coupling inertia coefficient
JT	Equivalent inertia coefficient of tank-fluid
KS	Static righting coefficient of ship
KT	Equivalent static coefficient of tank-fluid
LOGW	Natural logarithm of radian frequency
MAG	Magnitude of open-loop transfer function
MAGH	Magnitude of closed-loop transfer function
MDB	Decibel gain of open-loop transfer function
MDBH	Decibel gain of closed-loop transfer function
N	Numerator polynomial of transfer function
OLOOP	Open-loop transfer function
PHI	Phase of open-loop transfer function

Q	Denominator polynomial of open-loop transfer function
S	LaPlace transform variable
SST	Ship-tank response in feedback path

SUBROUTINES

SYMBOL	MEANING
A	Cross-sectional area of vertical leg of U-tube
AS	Cross-sectional area of horizontal leg of U-tube
BEAM	Transverse dimension of ship (beam)
BP	Velocity head-loss coefficient of pump
BS	Damping coefficient of ship
BT	Overall damping coefficient of tank-fluid
BT1	Skin-friction damping coefficient of tank fluid
D	Perpendicular distance from center of rotation of ship to horizontal cross-leg of U-tube
DABS	Absolute value (magnitude) of DCAL
DCAL	Calculated value of D to force the mutual-coupling inertia coefficient to zero
DISP	Displacement of ship
E1,E2	Incremental diffenence between current and previously calculated values of ship parameters
GM	Metacentric height
GRAV	Acceleration of gravity
H	Height of fluid column in vertical leg of U-tube
I1,I2,I3,I4	Evaluation of integrals as described in main body of thesis
JS	Roll inertia coefficient of ship

JS1	Roll inertia coefficient of unmodified ship
JST	Mutual-coupling inertia coefficient
JT	Equivalent inertia coefficient of tank-fluid
KRAD	Radius of gyration of the mass of the ship
KS	Static righting coefficient of ship
KS1	Static righting coefficient of unmodified ship
KT	Equivalent static coefficient of tank-fluid
L	Longitudinal dimension of a vertical leg of the U-tube
LEN	Longitudinal dimension of ship (length)
LMDA	Design ratio λ
N	Number of iterations of design procedure
R	Lever arm of U-tube
RHO	Mass density of tank-fluid
RHOG	Weight density of tank-fluid
TJ	Temporary JS
TK	Temporary KS
TPHI	Natural period of roll of ship
WIDE	Transverse dimension of vertical leg of U-tube
WN2	Natural frequency of ship-tank system, squared
X	Design ratio X

APPENDIX E

Waveslope Spectrum of the Ocean

The waveslope spectrum of the ocean was generated from the relationships of the trochoidal wave model. It is intended to be used in conjunction with the Pierson-Moskowitz wave power spectra.

This appendix includes the listing of the DSL/360 program which generated the spectrum, and listings of the spectra for sea states 3, 4, 5, and 6.

```
// EXEC DSL
//DSL.INPUT DD *
TITLE WAVESLOPE SPECTRUM OF THE OCEAN
TITLE SIGNIFICANT HEIGHT = 3.3 FEET
TITLE ********(SEA STATE 3)********
PARAM H=3.3
DYNAMIC
      LOGW=RAMP(0.)-1.
      W=10.**LCGW
      T=6.2831853/W
      L=5.118*T**2
      RATIO=L/H
      IF(RATIC.LE.7.)RATIO=7.
      PHIMAX=57.29578/RATIO
TERMINAL
CONTRL FINTIM=1.8,DELT=0.005,DELS=0.005
PRINT .025,W,PHIMAX
PREPAR .025,LCGW,PHIMAX
GRAPH LOGW,PHIMAX
LABEL WAVESLOPE SPECTRUM FOR H=3.3 FEET
PRPLOT ONLY
END
PARAM H=6.9
RESET TITLE
TITLE WAVESLOPE SPECTRUM OF THE OCEAN
TITLE SIGNIFICANT HEIGHT = 6.9 FEET
TITLE ********(SEA STATE 4)********
RESET LABEL
LABEL WAVESLCPE SPECTRUM FOR H=6.9 FEET
END
PARAM H=10.0
RESET TITLE
TITLE WAVESLOPE SPECTRUM OF THE OCEAN
TITLE SIGNIFICANT HEIGHT = 10. FEET
TITLE ********(SEA STATE 5)********
RESET LABEL
LABEL WAVESLCPE SPECTRUM FOR H=10.0 FEET
END
PARAM H=15.0
RESET TITLE
TITLE WAVESLOPE SPECTRUM OF THE OCEAN
TITLE SIGNIFICANT HEIGHT = 15. FEET
TITLE ********(SEA STATE 6)********
RESET LABEL
LABEL WAVESLCPE SPECTRUM FOR H=15.0 FEET
END
STOP
```

WAVESLOPE SPECTRUM OF THE OCEAN
SIGNIFICANT HEIGHT = 3.3 FEET
********(SEA STATE 3)********

W	PHIMAX
1.0000E-01	9.3579E-03
1.0593E-01	1.0500E-02
1.1220E-01	1.1781E-02
1.1885E-01	1.3218E-02
1.2589E-01	1.4831E-02
1.3335E-01	1.6641E-02
1.4125E-01	1.8671E-02
1.4962E-01	2.0950E-02
1.5849E-01	2.3506E-02
1.6788E-01	2.6374E-02
1.7783E-01	2.9592E-02
1.8836E-01	3.3203E-02
1.9953E-01	3.7254E-02
2.1135E-01	4.1800E-02
2.2387E-01	4.6900E-02
2.3714E-01	5.2623E-02
2.5119E-01	5.9044E-02
2.6607E-01	6.6249E-02
2.8184E-01	7.4332E-02
2.9854E-01	8.3402E-02
3.1623E-01	9.3579E-02
3.3497E-01	1.0500E-01
3.5481E-01	1.1781E-01
3.7584E-01	1.3218E-01
3.9811E-01	1.4831E-01
4.2170E-01	1.6641E-01
4.4668E-01	1.8671E-01
4.7315E-01	2.0950E-01
5.0119E-01	2.3506E-01
5.3088E-01	2.6374E-01
5.6234E-01	2.9592E-01
5.9566E-01	3.3203E-01
6.3096E-01	3.7254E-01
6.6834E-01	4.1800E-01
7.0794E-01	4.6900E-01
7.4989E-01	5.2623E-01
7.9433E-01	5.9044E-01
8.4139E-01	6.6248E-01
8.9125E-01	7.4332E-01
9.4406E-01	8.3402E-01
1.0000E 00	9.3578E-01
1.0593E 00	1.0500E 00
1.1220E 00	1.1781E 00
1.1885E 00	1.3218E 00
1.2589E 00	1.4831E 00
1.3335E 00	1.6641E 00
1.4125E 00	1.8671E 00
1.4962E 00	2.0949E 00
1.5849E 00	2.3506E 00
1.6788E 00	2.6374E 00
1.7783E 00	2.9592E 00
1.8836E 00	3.3203E 00
1.9953E 00	3.7254E 00

WAVESLOPE SPECTRUM OF THE OCEAN
SIGNIFICANT HEIGHT = 3.3 FEET
********(SEA STATE 3)********

W	PHIMAX
2.1135E 00	4.1800E 00
2.2387E 00	4.6900E 00
2.3714E 00	5.2623E 00
2.5119E 00	5.9044E 00
2.6607E 00	6.6248E 00
2.8184E 00	7.4331E 00
2.9854E 00	8.1851E 00
3.1623E 00	8.1851E 00
3.3496E 00	8.1851E 00
3.5481E 00	8.1851E 00
3.7584E 00	8.1851E 00
3.9811E 00	8.1851E 00
4.2169E 00	8.1851E 00
4.4668E 00	8.1851E 00
4.7315E 00	8.1851E 00
5.0118E 00	8.1851E 00
5.3088E 00	8.1851E 00
5.6234E 00	8.1851E 00
5.9566E 00	8.1851E 00
6.3095E 00	8.1851E 00

WAVESLOPE SPECTRUM OF THE OCEAN
SIGNIFICANT HEIGHT = 6.9 FEET
********(SEA STATE 4)********

W	PHIMAX
1.0000E-01	1.9566E-02
1.0593E-01	2.1954E-02
1.1220E-01	2.4633E-02
1.1885E-01	2.7638E-02
1.2589E-01	3.1011E-02
1.3335E-01	3.4795E-02
1.4125E-01	3.9040E-02
1.4962E-01	4.3804E-02
1.5849E-01	4.9149E-02
1.6788E-01	5.5146E-02
1.7783E-01	6.1875E-02
1.8836E-01	6.9424E-02
1.9953E-01	7.7895E-02
2.1135E-01	8.7400E-02
2.2387E-01	9.8065E-02
2.3714E-01	1.1003E-01
2.5119E-01	1.2346E-01
2.6607E-01	1.3852E-01
2.8184E-01	1.5542E-01
2.9854E-01	1.7439E-01
3.1623E-01	1.9566E-01
3.3497E-01	2.1954E-01
3.5481E-01	2.4633E-01
3.7584E-01	2.7638E-01
3.9811E-01	3.1011E-01
4.2170E-01	3.4795E-01
4.4668E-01	3.9040E-01
4.7315E-01	4.3804E-01
5.0119E-01	4.9149E-01
5.3088E-01	5.5146E-01
5.6234E-01	6.1874E-01
5.9566E-01	6.9424E-01
6.3096E-01	7.7895E-01
6.6834E-01	8.7400E-01
7.0794E-01	9.8064E-01
7.4989E-01	1.1003E 00
7.9433E-01	1.2346E 00
8.4139E-01	1.3852E 00
8.9125E-01	1.5542E 00
9.4406E-01	1.7439E 00
1.0000E 00	1.9566E 00
1.0593E 00	2.1954E 00
1.1220E 00	2.4633E 00
1.1885E 00	2.7638E 00
1.2589E 00	3.1011E 00
1.3335E 00	3.4794E 00
1.4125E 00	3.9040E 00
1.4962E 00	4.3803E 00
1.5849E 00	4.9148E 00
1.6788E 00	5.5145E 00
1.7783E 00	6.1874E 00
1.8836E 00	6.9424E 00
1.9953E 00	7.7895E 00

WAVESLOPE SPECTRUM OF THE OCEAN
SIGNIFICANT HEIGHT = 6.9 FEET
********(SEA STATE 4)********

W	PHIMAX
2.1135E 00	8.1851E 00
2.2387E 00	8.1851E 00
2.3714E 00	8.1851E 00
2.5119E 00	8.1851E 00
2.6607E 00	8.1851E 00
2.8184E 00	8.1851E 00
2.9854E 00	8.1851E 00
3.1623E 00	8.1851E 00
3.3496E 00	8.1851E 00
3.5481E 00	8.1851E 00
3.7584E 00	8.1851E 00
3.9811E 00	8.1851E 00
4.2169E 00	8.1851E 00
4.4668E 00	8.1851E 00
4.7315E 00	8.1851E 00
5.0118E 00	8.1851E 00
5.3088E 00	8.1851E 00
5.6234E 00	8.1851E 00
5.9566E 00	8.1851E 00
6.3095E 00	8.1851E 00

WAVESLOPE SPECTRUM OF THE OCEAN
SIGNIFICANT HEIGHT = 10. FEET
********(SEA STATE 5)********

W	PHIMAX
1.0000E-01	2.8357E-02
1.0593E-01	3.1817E-02
1.1220E-01	3.5700E-02
1.1885E-01	4.0056E-02
1.2589E-01	4.4943E-02
1.3335E-01	5.0427E-02
1.4125E-01	5.6580E-02
1.4962E-01	6.3484E-02
1.5849E-01	7.1230E-02
1.6788E-01	7.9921E-02
1.7783E-01	8.9673E-02
1.8836E-01	1.0062E-01
1.9953E-01	1.1289E-01
2.1135E-01	1.2667E-01
2.2387E-01	1.4212E-01
2.3714E-01	1.5946E-01
2.5119E-01	1.7892E-01
2.6607E-01	2.0075E-01
2.8184E-01	2.2525E-01
2.9854E-01	2.5273E-01
3.1623E-01	2.8357E-01
3.3497E-01	3.1817E-01
3.5481E-01	3.5699E-01
3.7584E-01	4.0056E-01
3.9811E-01	4.4943E-01
4.2170E-01	5.0427E-01
4.4668E-01	5.6580E-01
4.7315E-01	6.3484E-01
5.0119E-01	7.1230E-01
5.3088E-01	7.9921E-01
5.6234E-01	8.9673E-01
5.9566E-01	1.0061E 00
6.3096E-01	1.1289E 00
6.6834E-01	1.2667E 00
7.0794E-01	1.4212E 00
7.4989E-01	1.5946E 00
7.9433E-01	1.7892E 00
8.4139E-01	2.0075E 00
8.9125E-01	2.2525E 00
9.4406E-01	2.5273E 00
1.0000E 00	2.8357E 00
1.0593E 00	3.1817E 00
1.1220E 00	3.5699E 00
1.1885E 00	4.0055E 00
1.2589E 00	4.4943E 00
1.3335E 00	5.0427E 00
1.4125E 00	5.6580E 00
1.4962E 00	6.3483E 00
1.5849E 00	7.1230E 00
1.6788E 00	7.9921E 00
1.7783E 00	8.1851E 00
1.8836E 00	8.1851E 00
1.9953E 00	8.1851E 00

WAVESLOPE SPECTRUM OF THE OCEAN
SIGNIFICANT HEIGHT = 10. FEET
********(SEA STATE 5)********

W	PHIMAX
2.1135E 00	8.1851E 00
2.2387E 00	8.1851E 00
2.3714E 00	8.1851E 00
2.5119E 00	8.1851E 00
2.6607E 00	8.1851E 00
2.8184E 00	8.1851E C0
2.9854E 00	8.1851E 00
3.1623E 00	8.1851E 00
3.3496E 00	8.1851E 00
3.5481E 00	8.1851E 00
3.7584E 00	8.1851E 00
3.9811E 00	8.1851E 00
4.2169E 00	8.1851E 00
4.4668E 00	8.1851E 00
4.7315E 00	8.1851E 00
5.0118E 00	8.1851E 00
5.3088E 00	8.1851E 00
5.6234E 00	8.1851E C0
5.9566E 00	8.1851E 00
6.3095E 00	8.1851E 00

WAVESLOPE SPECTRUM OF THE OCEAN
SIGNIFICANT HEIGHT = 15. FEET
********(SEA STATE 6)********

W	PHIMAX
1.0000E-01	4.2536E-02
1.0593E-01	4.7726E-02
1.1220E-01	5.3549E-02
1.1885E-01	6.0083E-02
1.2589E-01	6.7415E-02
1.3335E-01	7.5641E-02
1.4125E-01	8.4870E-02
1.4962E-01	9.5226E-02
1.5849E-01	1.0685E-01
1.6788E-01	1.1988E-01
1.7783E-01	1.3451E-01
1.8836E-01	1.5092E-01
1.9953E-01	1.6934E-01
2.1135E-01	1.9000E-01
2.2387E-01	2.1318E-01
2.3714E-01	2.3920E-01
2.5119E-01	2.6838E-01
2.6607E-01	3.0113E-01
2.8184E-01	3.3787E-01
2.9854E-01	3.7910E-01
3.1623E-01	4.2536E-01
3.3497E-01	4.7726E-01
3.5481E-01	5.3549E-01
3.7584E-01	6.0083E-01
3.9811E-01	6.7414E-01
4.2170E-01	7.5640E-01
4.4668E-01	8.4870E-01
4.7315E-01	9.5225E-01
5.0119E-01	1.0684E 00
5.3088E-01	1.1988E 00
5.6234E-01	1.3451E 00
5.9566E-01	1.5092E 00
6.3096E-01	1.6934E 00
6.6834E-01	1.9000E 00
7.0794E-01	2.1318E 00
7.4989E-01	2.3920E 00
7.9433E-01	2.6838E 00
8.4139E-01	3.0113E 00
8.9125E-01	3.3787E 00
9.4406E-01	3.7910E 00
1.0000E 00	4.2536E 00
1.0593E 00	4.7726E 00
1.1220E 00	5.3549E 00
1.1885E 00	6.0083E 00
1.2589E 00	6.7414E 00
1.3335E 00	7.5640E 00
1.4125E 00	8.1851E 00
1.4962E 00	8.1851E 00
1.5849E 00	8.1851E 00
1.6788E 00	8.1851E 00
1.7783E 00	8.1851E 00
1.8836E 00	8.1851E 00
1.9953E 00	8.1851E 00

WAVESLOPE SPECTRUM OF THE OCEAN
SIGNIFICANT HEIGHT = 15. FEET
********(SEA STATE 6)********

W	PHIMAX
2.1135E 00	8.1851E 00
2.2387E 00	8.1851E 00
2.3714E 00	8.1851E 00
2.5119E 00	8.1851E 00
2.6607E 00	8.1851E 00
2.8184E 00	8.1851E 00
2.9854E 00	8.1851E 00
3.1623E 00	8.1851E 00
3.3496E 00	8.1851E 00
3.5481E 00	8.1851E 00
3.7584E 00	8.1851E 00
3.9811E 00	8.1851E 00
4.2169E 00	8.1851E 00
4.4668E 00	8.1851E 00
4.7315E 00	8.1851E 00
5.0118E 00	8.1851E 00
5.3088E 00	8.1851E 00
5.6234E 00	8.1851E 00
5.9566E 00	8.1851E 00
6.3095E 00	8.1851E 00

BIBLIOGRAPHY

Anastassakis, Anthony Nikolaos, Computer Aided Design of Optimal Compensators, MSEE Thesis, Naval Postgraduate School, Monterey, CA, December, 1977.

Blagoveshchensky, S. N., Theory of Ship Motions, translated from first Russian edition by Theodor and Leonilla Strelkoff, Louis Landweber(editor), Iowa Institute of Hydraulic Research, State University of Iowa. In two volumes, New York: Dover Publications, Inc., 1962.

Chadwick, Joseph H., The Anti-Roll Stabilization of Ships by means of Activated Tanks - Part A: Formulation of Problem and Basic Theory, Technical Report No. 15 under Contract N6-ONR-251 Task Order 2 (NR-041-943). Stanford, CA: Division of Engineering Mechanics, Stanford University, December 1950.

Chadwick, Joseph H. Jr., and Klotter, Karl, On the Dynamics of Anti-Rolling Tanks - Supplement to Technical Report No. 15, N6-ONR-251-2, Tecnical Report No. 2 under Contract N6-ONR-25129 (NR-041-114). Stanford,CA: Department of Electrical Engineering, Stanford University, 16 February 1953.

Comstock, John Paul (editor), Principles of Naval Architecture. New York: Society of Naval Architects and Marine Engineers (SNAME), 1967, including errata through December 1970.

Gillmer, Thomas C, Fundamentals of Construction and Stability of Naval Ships. Annapolis,MD: U.S. Naval Institute, 1969.

Inoue, Tokujiro, On the Growth of the Spectrum of a Wind Generated Sea According to a Modified Miles-Phillips Mechanism, Technical Report TR 66-6 under Contract N62306-1589, Task Order 3. Bronx, New York, NY: School of Engineering and Science, Research Division, New York University, April 1966.

----Ocean Wave Spectra, Proceedings of a conference sponsored by the U.S. Naval Oceanographic Office and the Division of Earth Sciences, National Academy of Sciences, National Research Council, Easton, Maryland, 1-4 May 1961. Englewood Cliffs, NJ: Prentice-Hall, 1963.

Vachanaratana, Manop, Design on the Frequency Parameter Plane with Application to Roll Stabilization of Ship, MSEE Thesis, Naval Postgraduate School, Monterey, CA, December 1973.

Ziegler, Phillip Eugene, Anti-Rolling Tank Design, MSME Thesis, Massachusetts Institute of Technology, Boston, MA, May 1975.

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